Proximate deconfined quantum critical point in $SrCu_2(BO_3)_2$

Lu Liu

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Science, Yi Cui, **Lu Liu**, Huihang Lin *et al.* (2023)

Outline

- Introduction: deconfined quantum critical point (DQCP)
- $SrCu_2(BO_3)_2$ (SCBO), Shastry-Sutherland model (SSM),
  Checkerboard J-Q model (CBJQ)
- NMR results for $SrCu_2(BO_3)_2$
  evidence for AFM and VBS states
  evidence for proximate DQCP
- Numerical results for CBJQ model with magnetic field
- Conclusion

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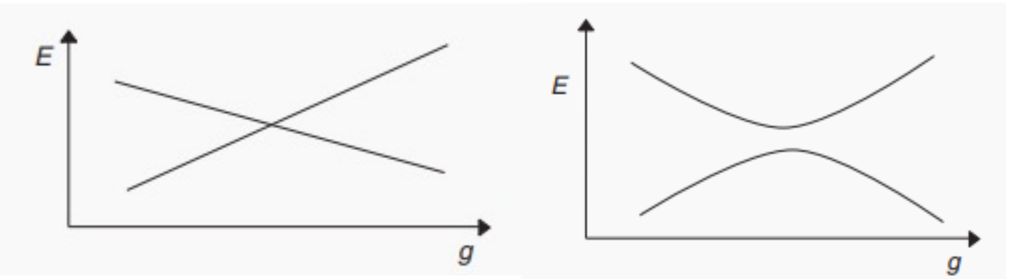
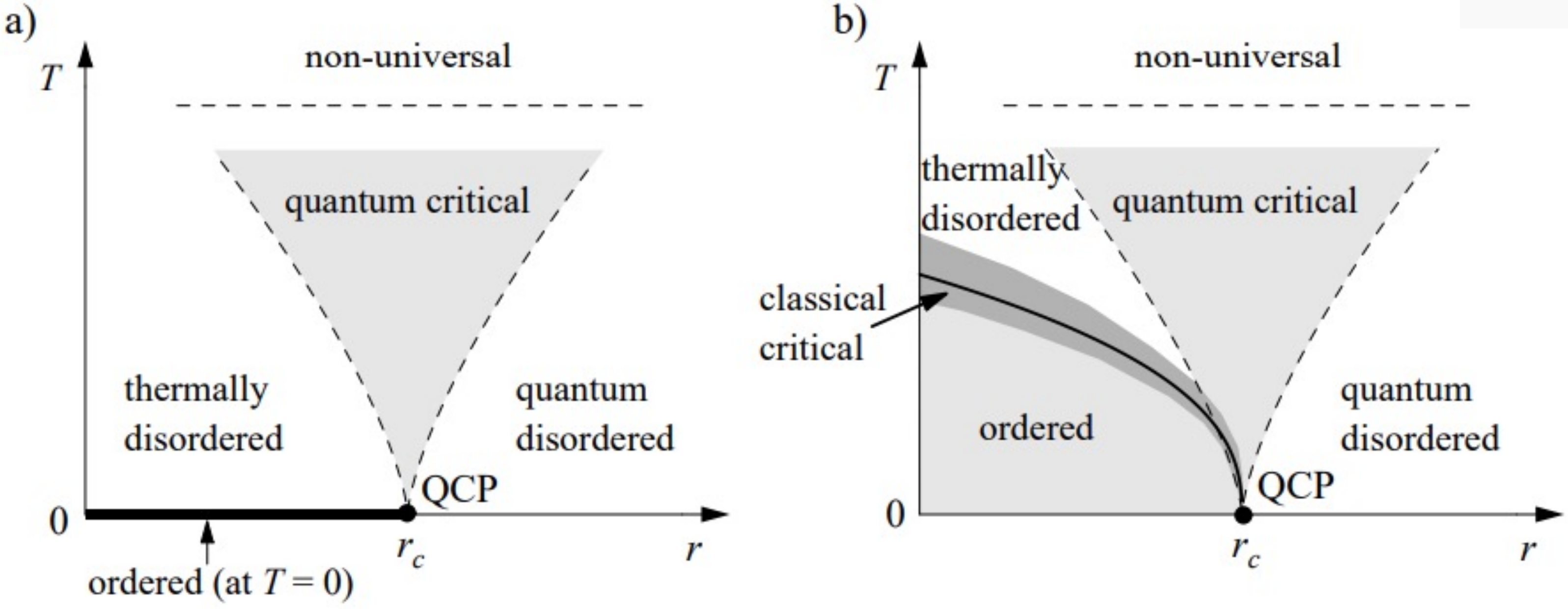
Classical phase transition: driven by thermal fluctuations

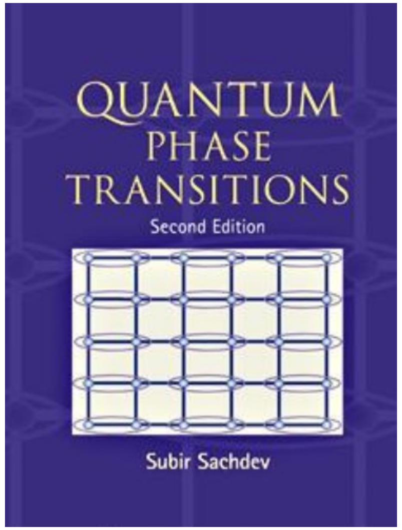
Quantum phase transition (QPT): a phase transition at zero temperature by tuning one parameter (in the Hamiltonian), driven by quantum fluctuations

- A quantum phase transition can be first-order or continuous
- Quantum critical point (QCP): a continuous QPT.
- Emergent symmetries can occur at QCP, e.g. SO(5) symmetry





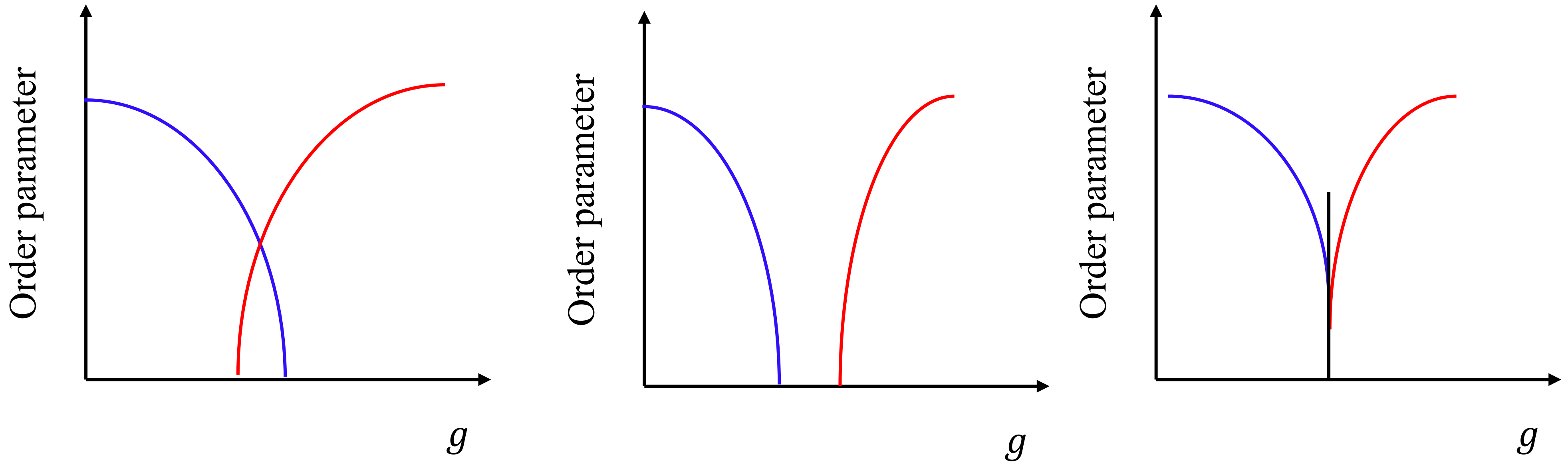
Rep. Prog. Phys. M. Vojta 2003



Landau-Ginzburg-Wilson (LGW) paradigm： use the order parameter to description the phase transition
the effective field theory are described by order parameter.

$$F[\psi(r)] = \int A|\nabla\psi|^2 + \alpha|\psi|^2 + \beta|\psi|^4 \, d^D r \qquad \alpha = \alpha_0(T - T_c) \qquad \psi \textit{ is the order parameter}$$

What about phase transition between two broken symmetry phase and the symmetry of each phase is not the subgroup of the other?

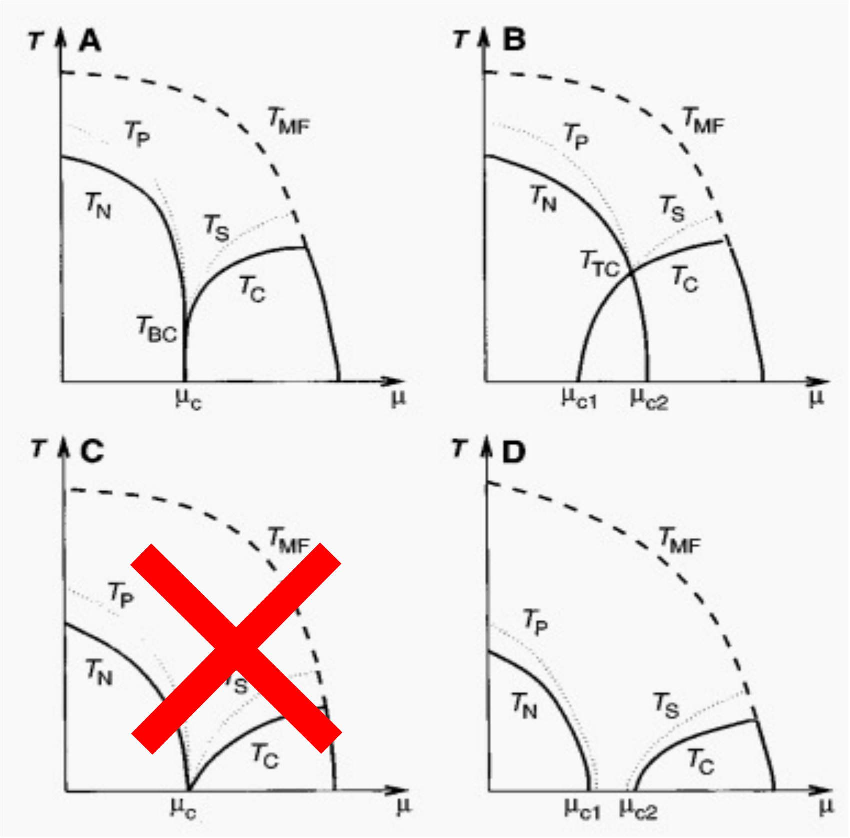


Direct phase transition between these two phases should be first-order transition or requires fine-tuning to a multicritical point to realize the second-order phase transition

SC AFM

according to the Landau theory, a direct continuous transition between SC and AFM is not allowed

A. Direct first-order transition that terminates at a bicritical point T_{bc}
B. Two second-order phase transition with an intermediate spin-bag phase
C. A single second-order phase transition at a quantum critical point
D. Two second-order quantum phase transition with an intermediate quantum -disorder phase.



Shou-cheng Zhang, Science (1997)

DQCP (Senthil, Vishwanath, Balents, Sachdev, Fisher, Science 300 1490 (2004))

Neel-VBS (valence bond solids)

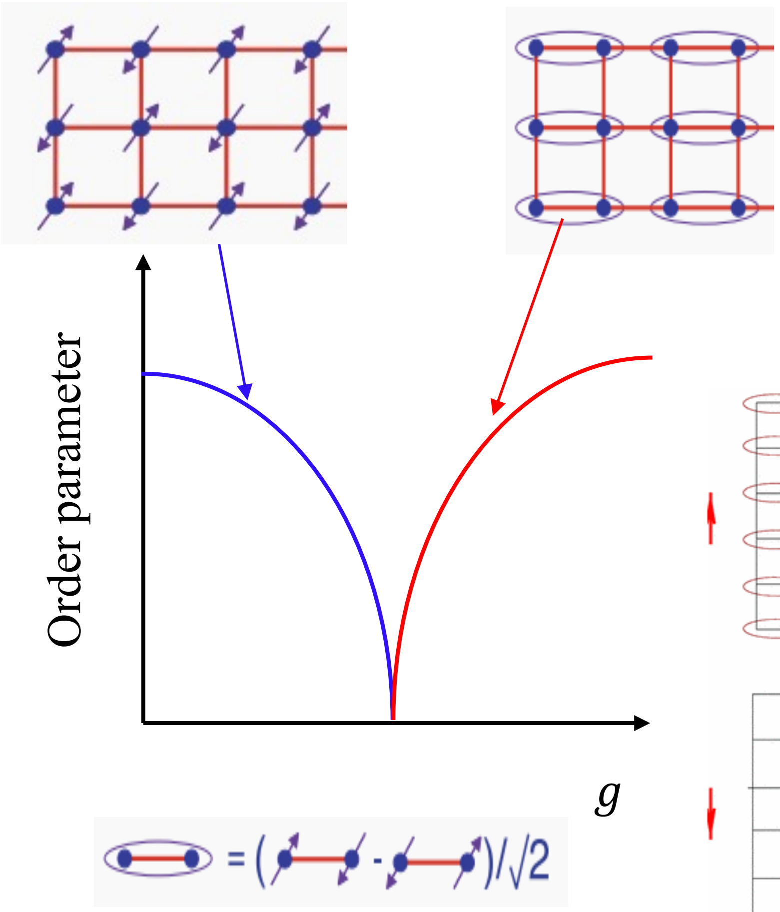
Continue phase transition, beyond LGW theory

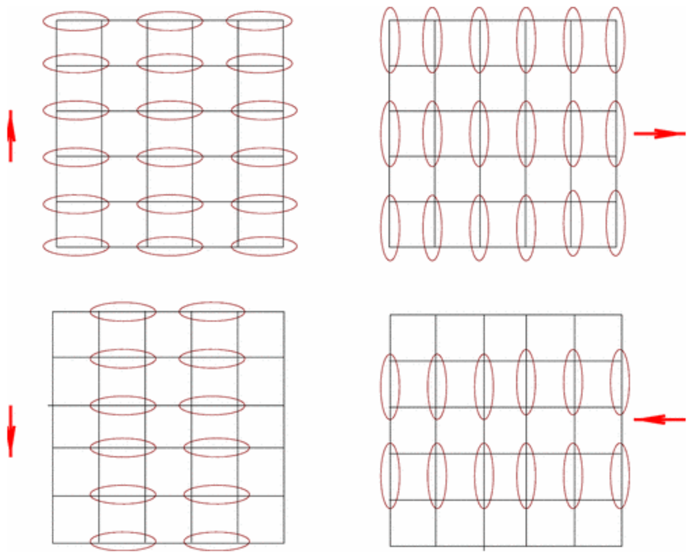
Field theory: $NCCP^1$

Fractionalized degree of freedom: spinon

Emergent gauge field: U(1)

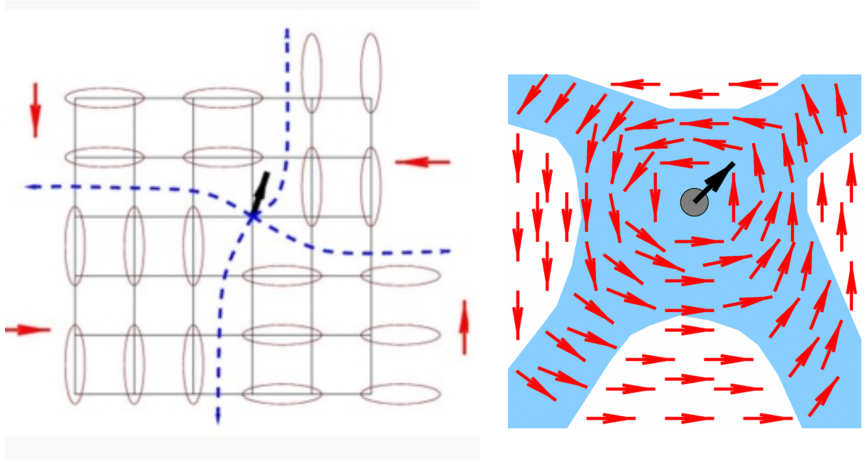
Topological defects are suppressed and conservation of skyrmion





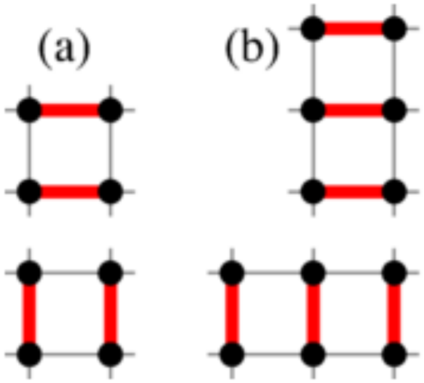
Deconfined: fractionalized degree of freedom

Topological defect of one phase carries the quantum number of the other phase



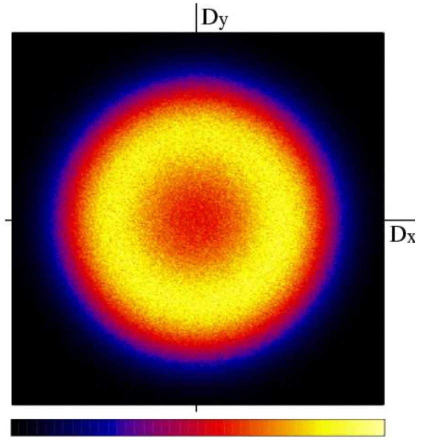
M. Levin, T. Senthil, PRB 70 220403 (2004)

Models for DQCP

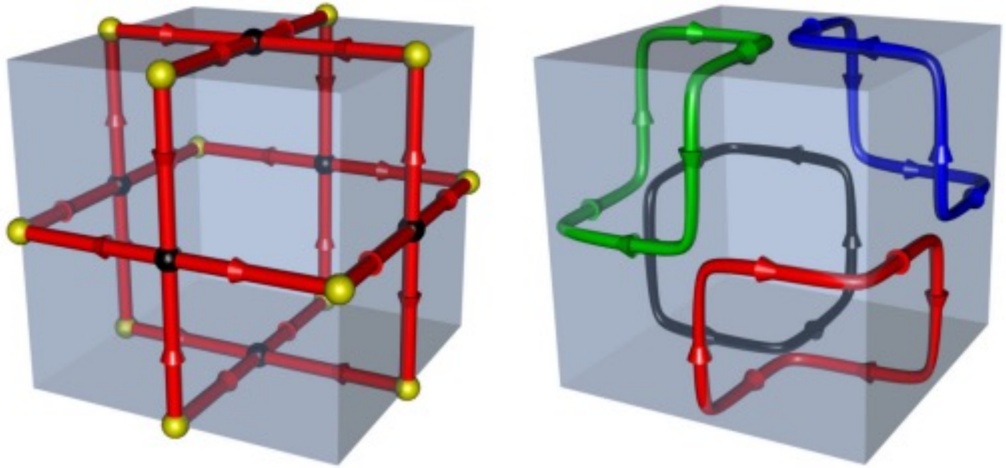


$$H = -\sum_{b=1}^{2N} J_b P_b - Q \sum_{p=1}^{2N} \prod_{\{b_p\}} P_{b_p}$$

$$P_b = P_{ij} = 1/4 - \mathbf{S}_j \cdot \mathbf{S}_j$$

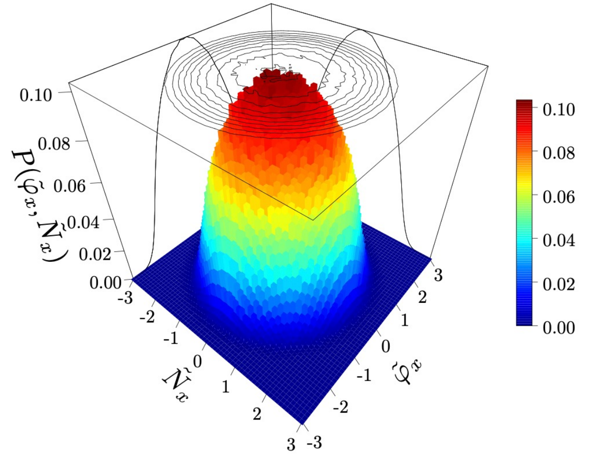


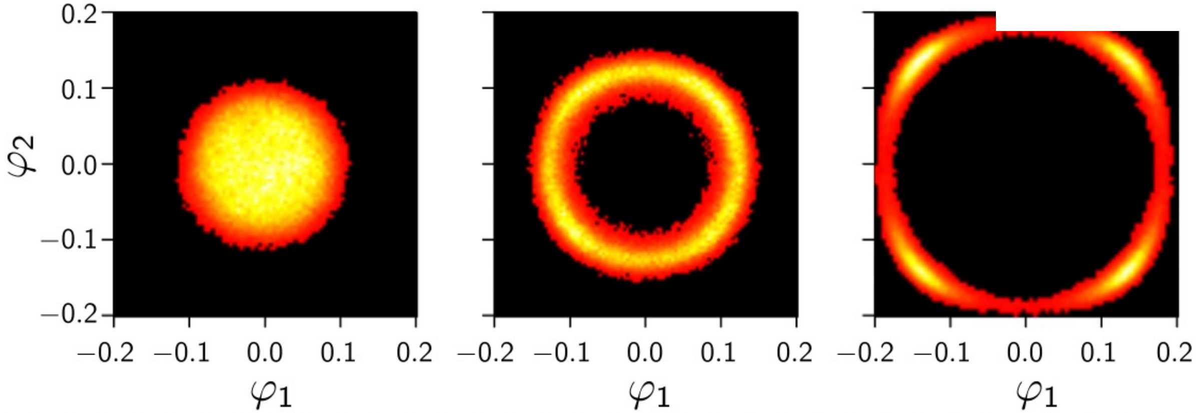
J-Q model, A. W. Sandvik, PRL (2007)



$$Z = \sum_{\substack{\text{colored} \\ \text{loop configs}}} \exp(-E),$$

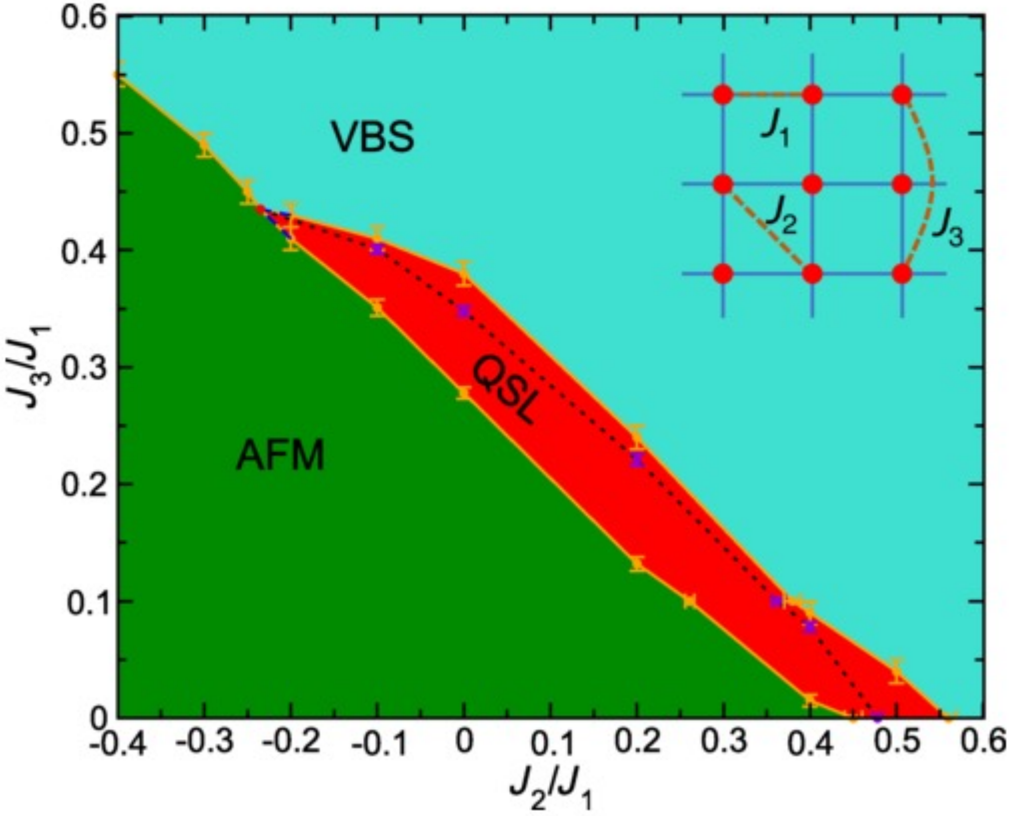
$$E = -J\left(\sum_{\langle r,r'\rangle \in A} \sigma_r \sigma_{r'} + \sum_{\langle r,r'\rangle \in B} \sigma_r \sigma_{r'}\right).$$



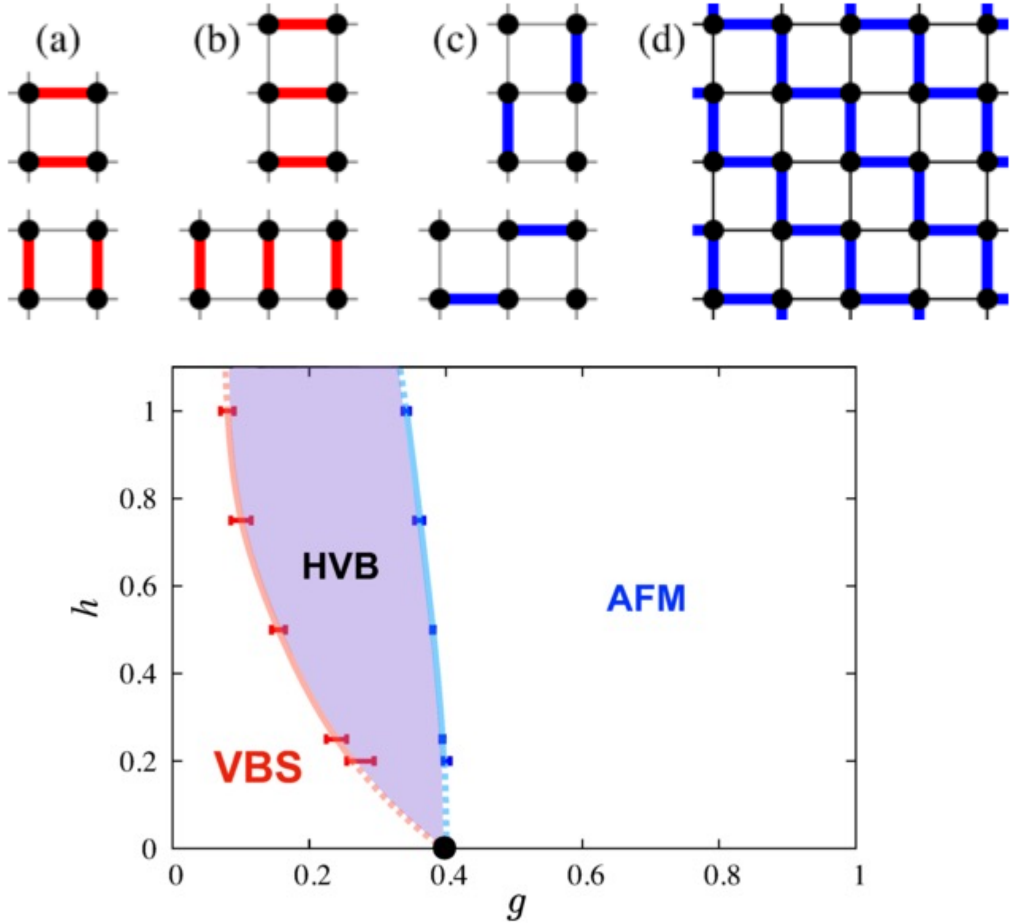


Classical loop model, A. Nahum, PRX (2015)
A. Nahum, PRL (2015)

Alternative suggestions: multicritical point for DQCP



W. Liu et al. PRX 12 031039 (2022)
tensor network method



HVB (helical valence-bond)

B. Zhao et al. PRL 125 257204 (2020)

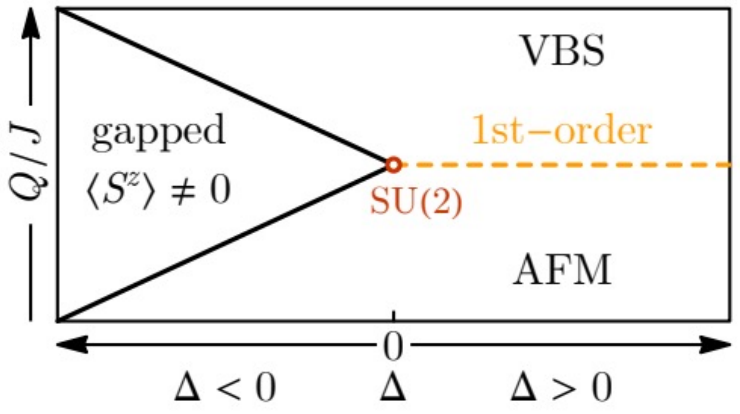


FIG. 6. Schematic phase diagram of the easy-plane J-Q model Eq. (22).

D.-C. Lu, Phys. Rev. B 104, 205142 (2021)

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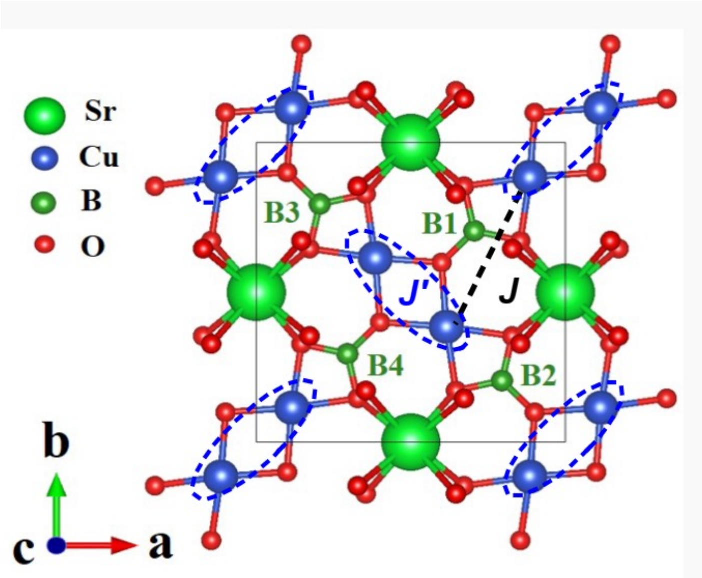
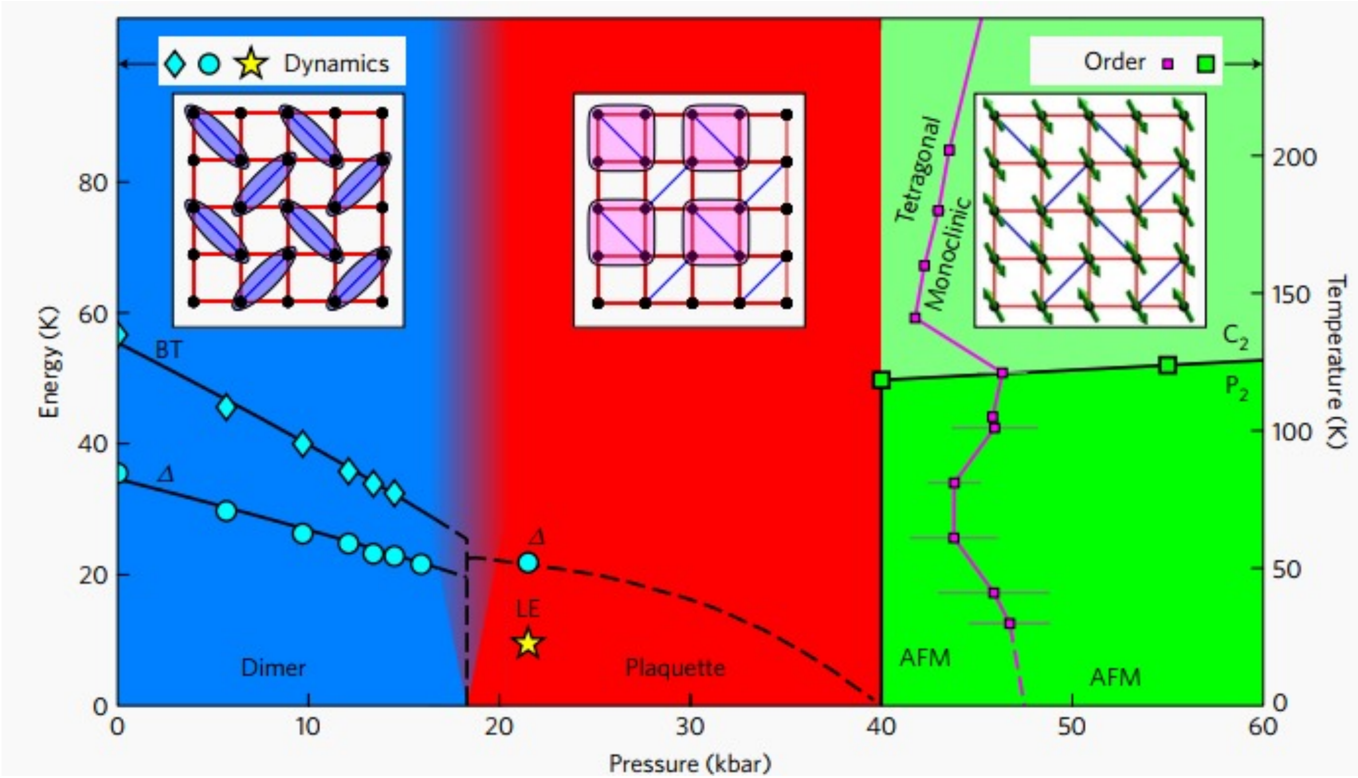
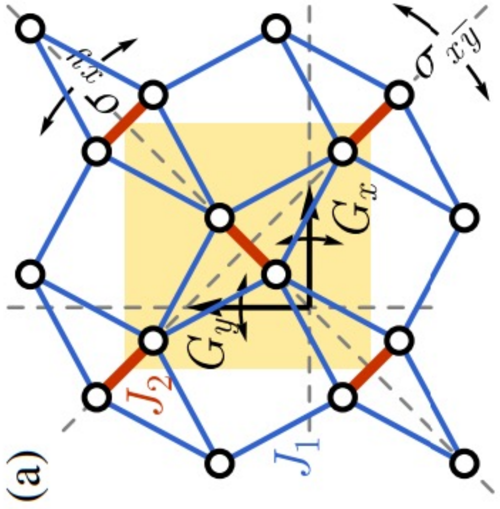
$SrCu_2(BO_3)_2$

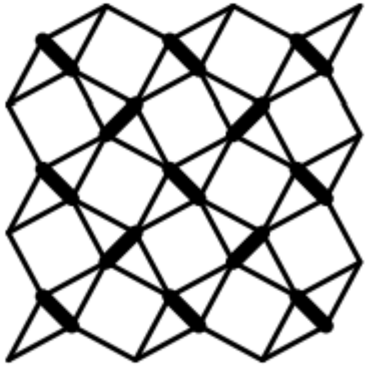
M. E. Zayed et al. (2017) reported a potential experimental realization of DQCP was reported in the quasi-2D Shastry–Sutherland compound $SrCu_2(BO_3)_2$ under pressure

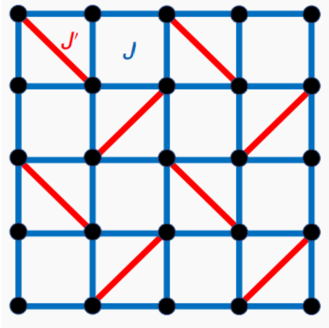
DS (dimer singlet) PS (plaquette singlet) AFM



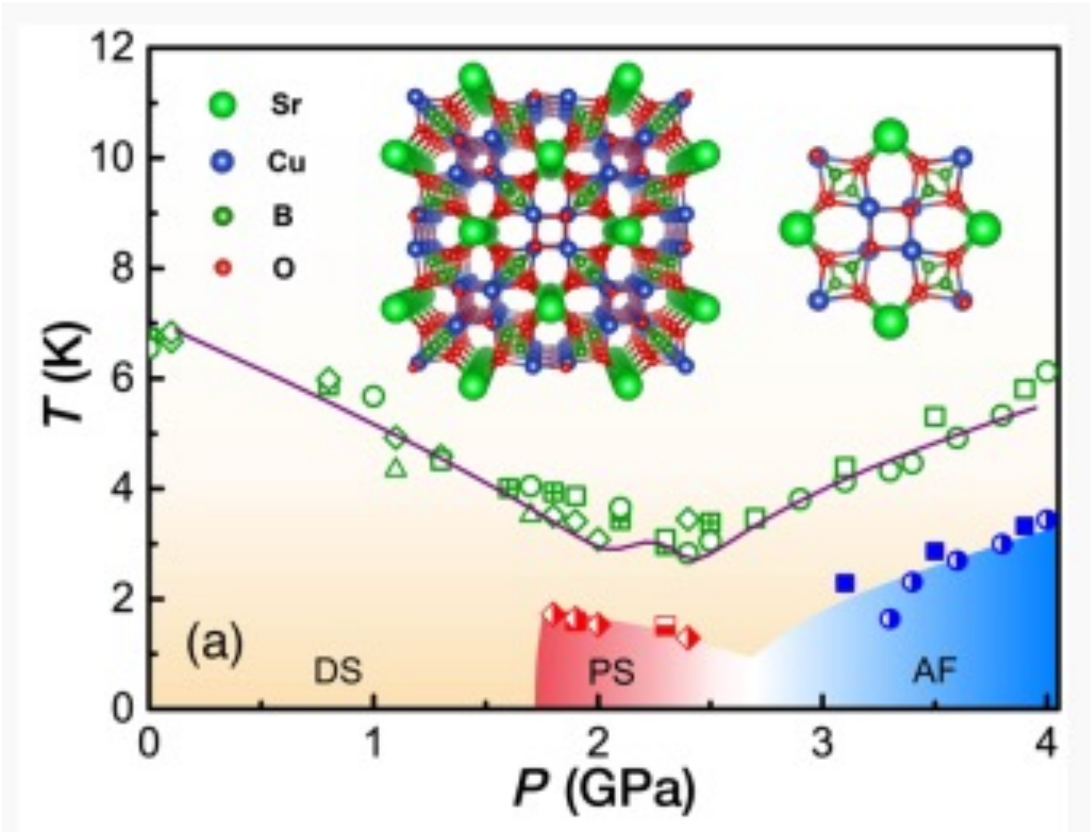


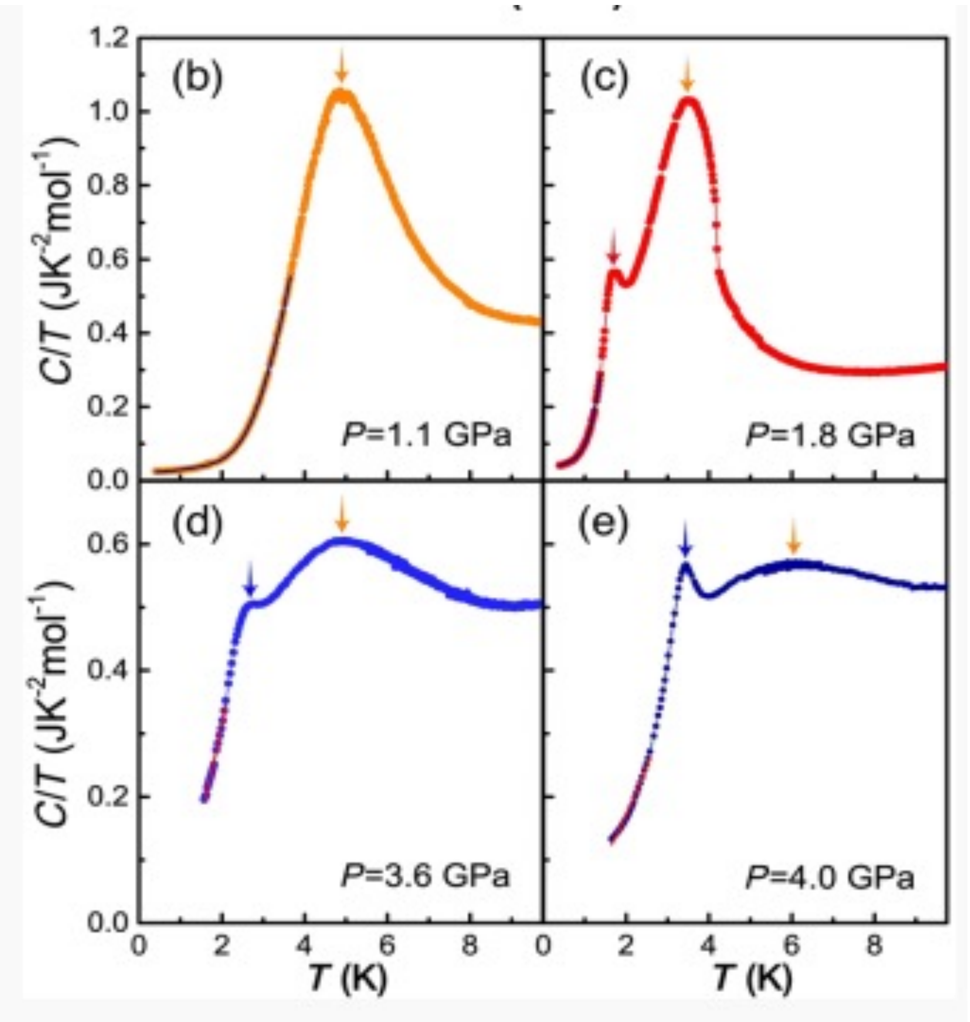






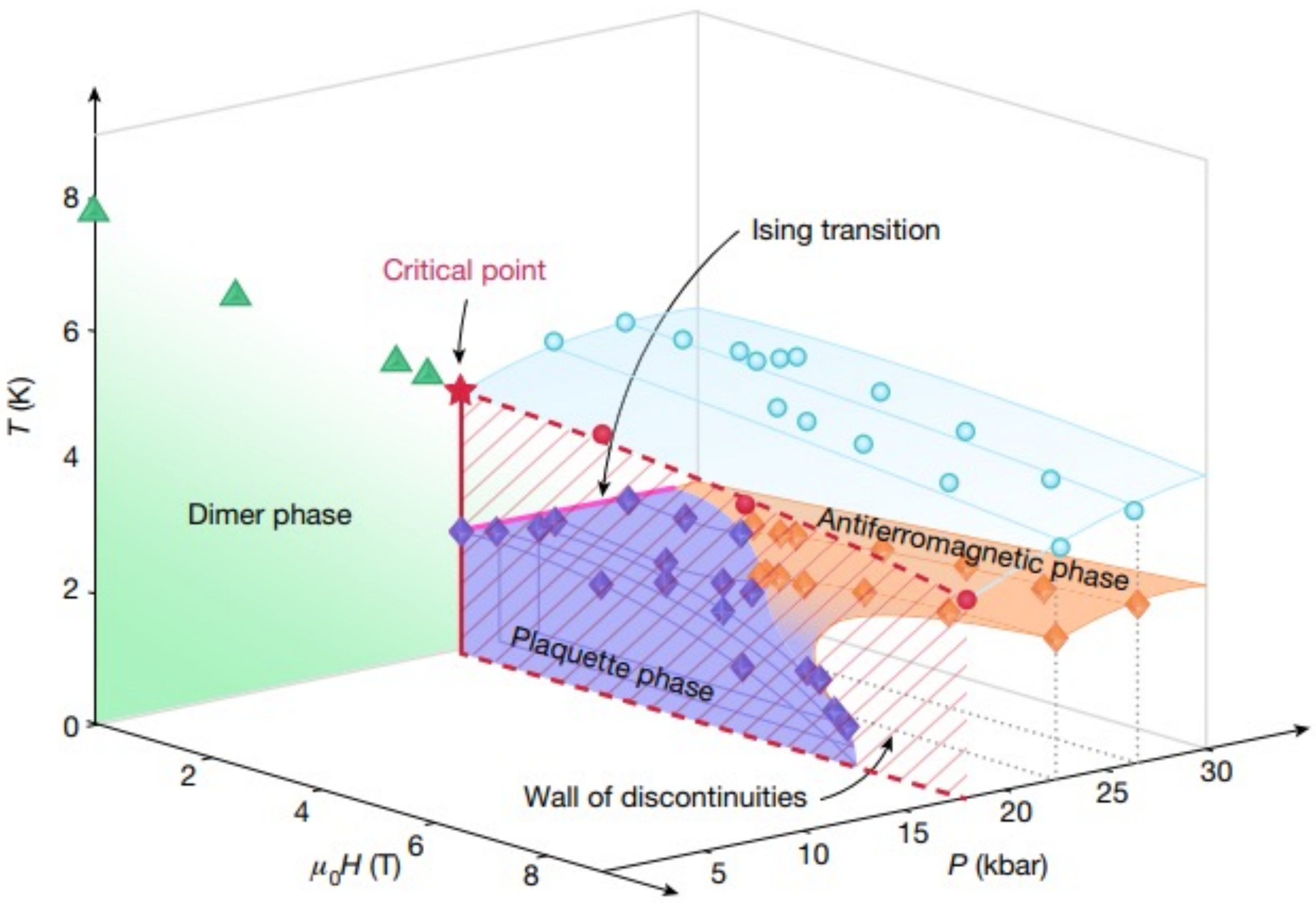
M. E. Zayed el al. Nat. Phys. 13 962 (2017)





DS (dimer singlet) PS (plaquette-singlet) AFM

J. Guo *et al.,* PRL 124 206602 (2020)



J. M. Jiménez, Nature 592 370 (2021)

Shastry-Sutherland model (SSM)

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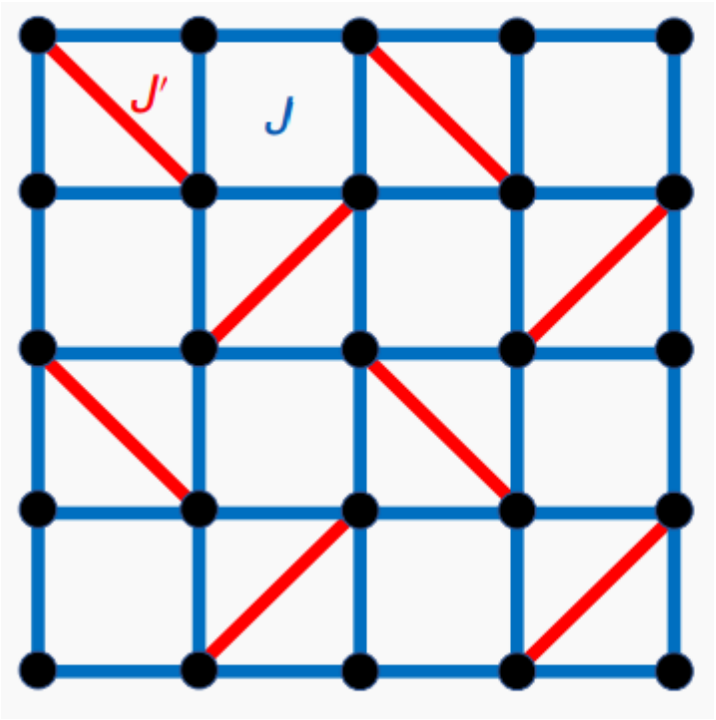
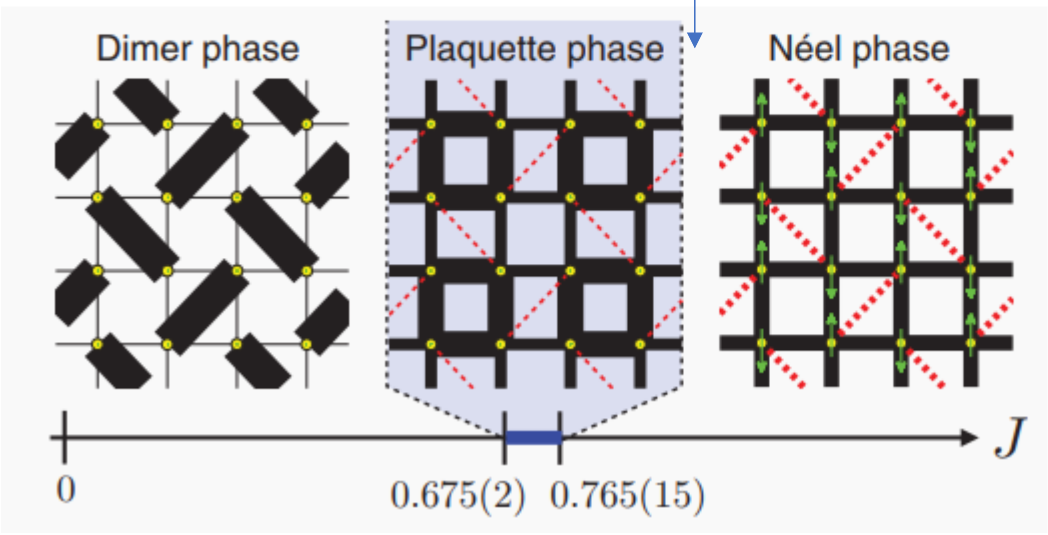


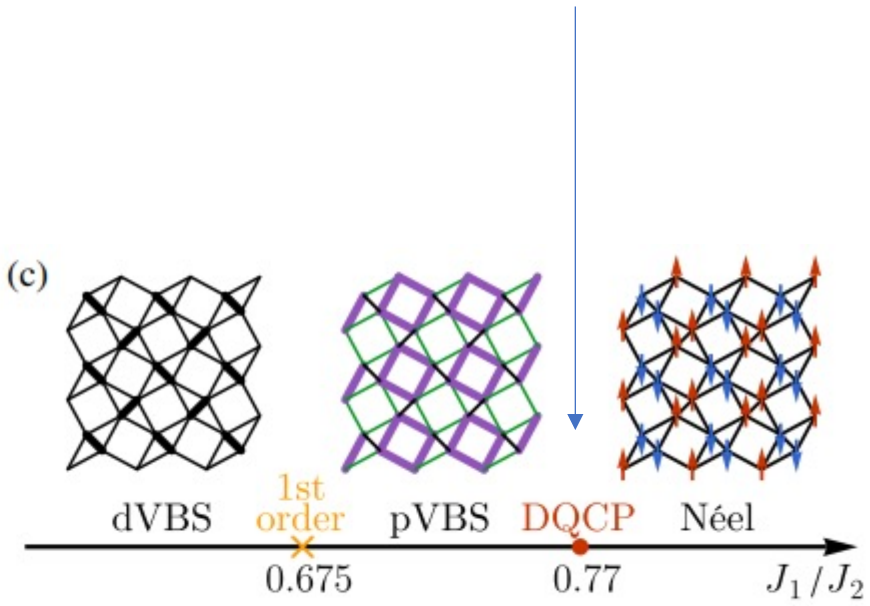
Shastry, Sutherland, Physica B 108 1069 (1981)

Tensor network: weakly first-order



Corboz, Mila, PRB 87 115114 (2013)

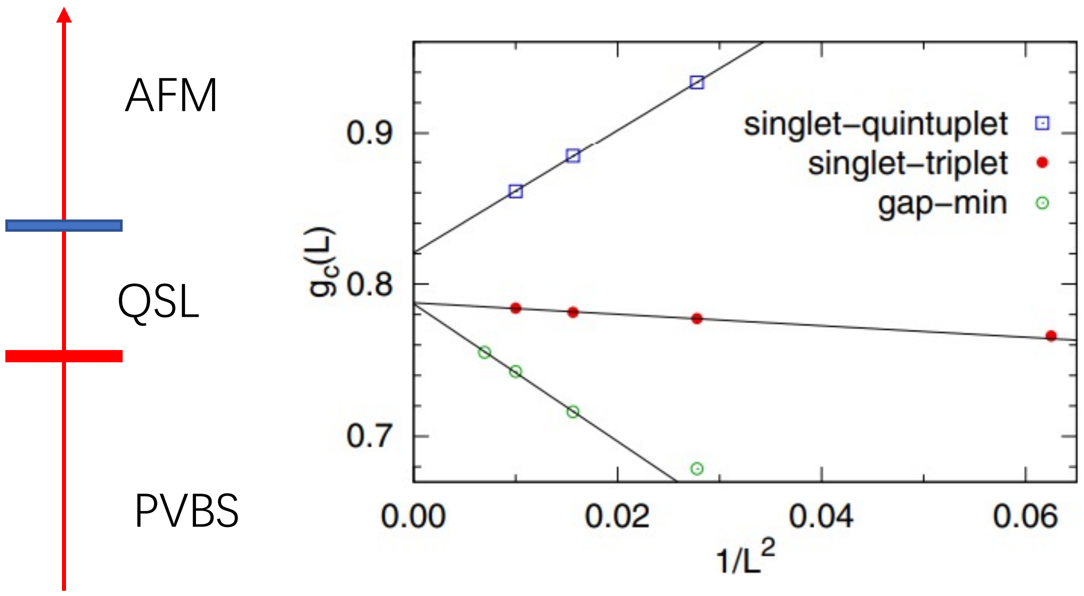
DMRG: DQCP between Neel and PVBS
emergent O(4) symmetry



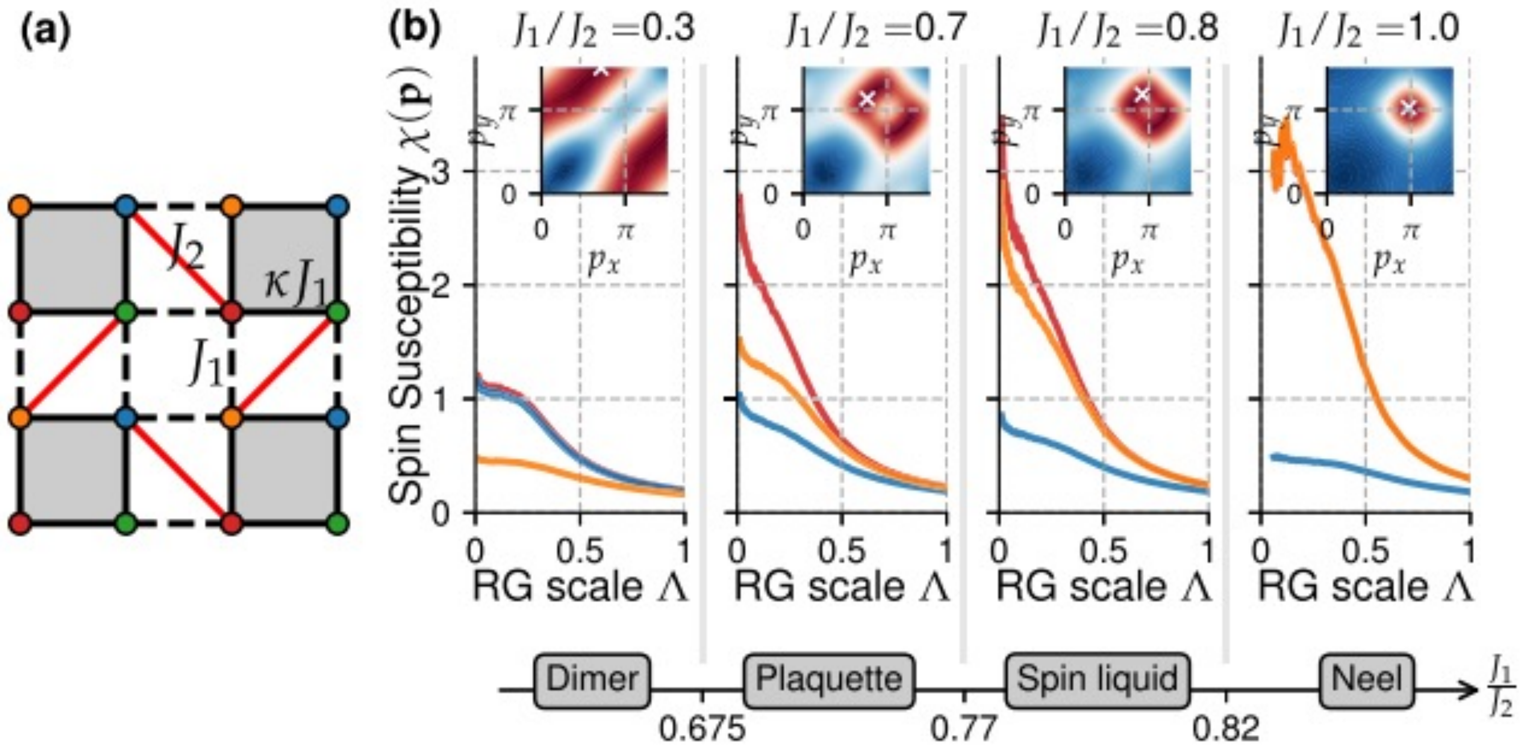
J. Y. Lee, Y. You, S. Sachdev, A. Vishwanath
PRX 9 041037 (2019)

DMRG

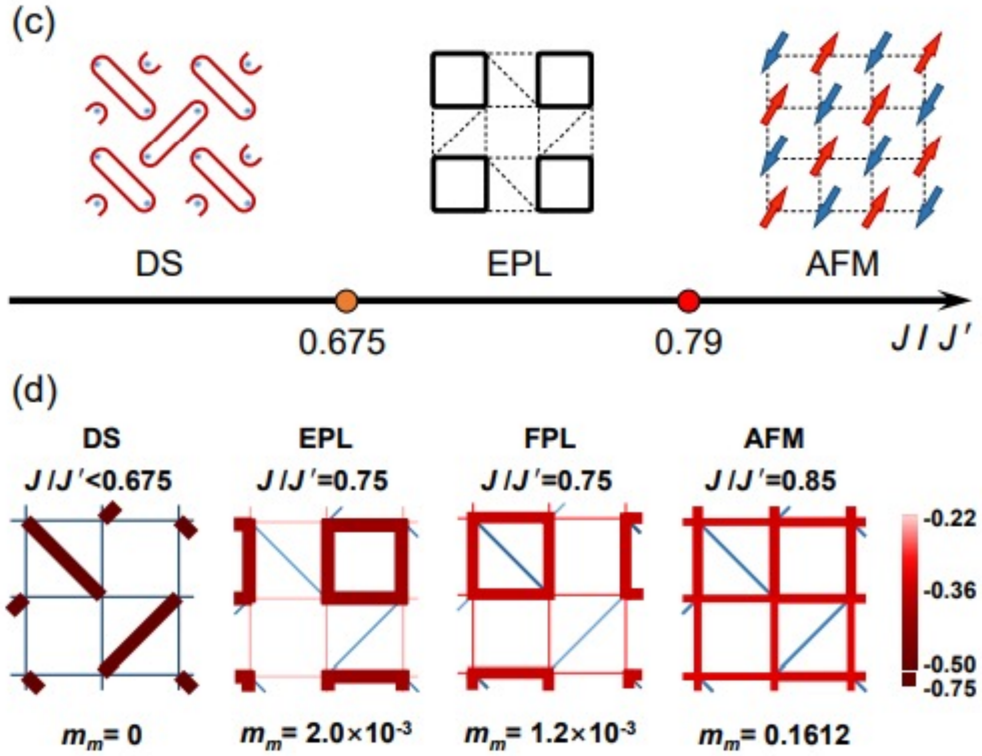
$g = J/J'$

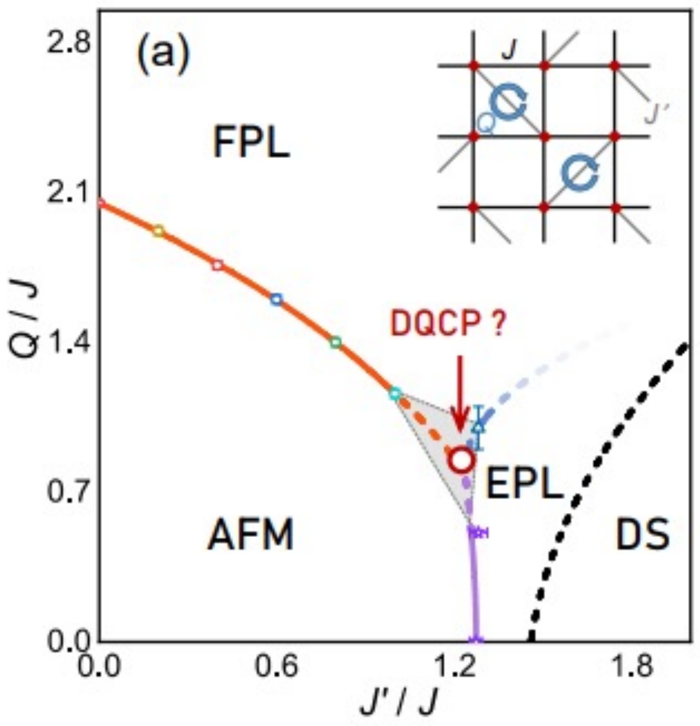


J. Yang, A. W. Sandvik , L. Wang
PRB 105 L060409 (2022)



Ahmet Keles, Erhai Zhao, PRB 105 L041115 (2022)
pseudofermion functional renormalization group

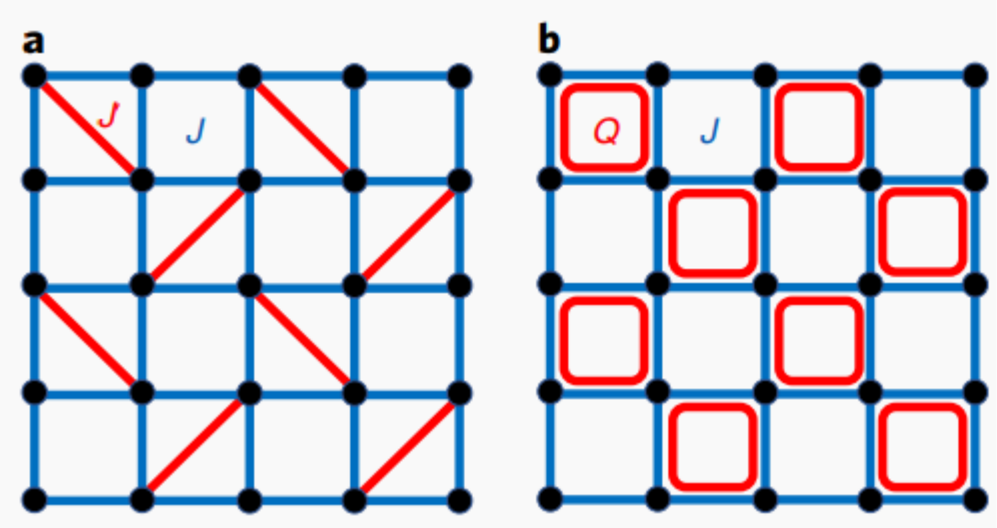




N. Xi *et al.* arXiv:2111. 07368 (2021)

variational optimization of the infinite tensor network states

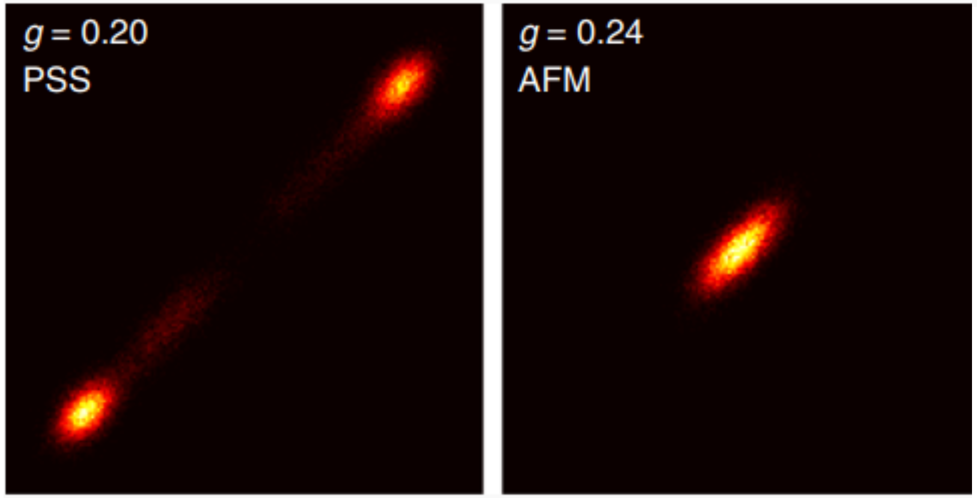
Checkerboard J-Q model
(CBJQ)
Sign problem free



$$H = -J\sum_{\langle ij\rangle} P_{ij} - Q\sum_{ijkl\in\square'} (P_{ij}P_{kl} + P_{ik}P_{jl})$$

$$P_{ij} = (1/4 - \mathbf{S}_i\cdot\mathbf{S}_j)$$

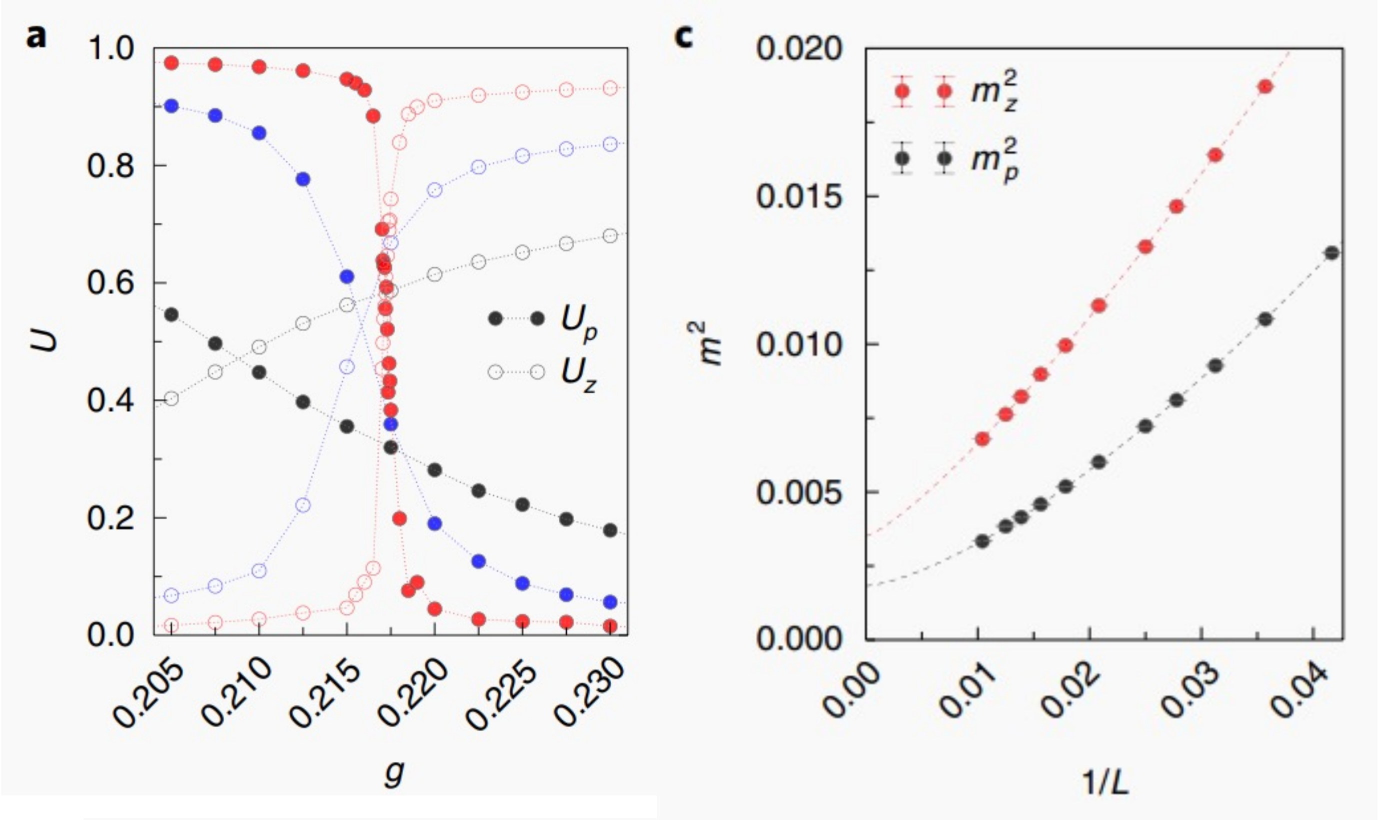
$$D_\mu = \frac{1}{L^2}\sum_{\mathbf{r}} (-1)^{r_\mu}\mathbf{S}(\mathbf{r})\cdot\mathbf{S}(\mathbf{r}+\hat{\mu}), \quad \hat{\mu} = \hat{x}, \hat{y}$$

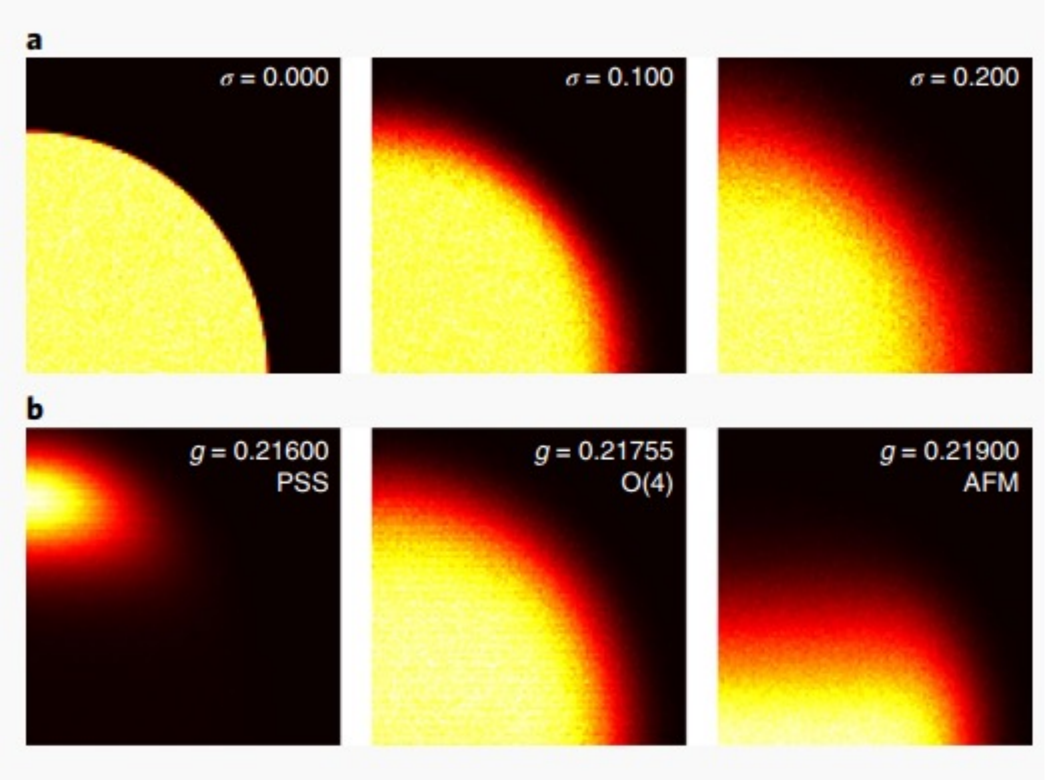


AFM-PSS (plaquette-singlet solid)

with $g_c = \frac{J}{Q} = 0.2175$

Bowen Zhao et al. Nat. Phys. 15 678 (2019)





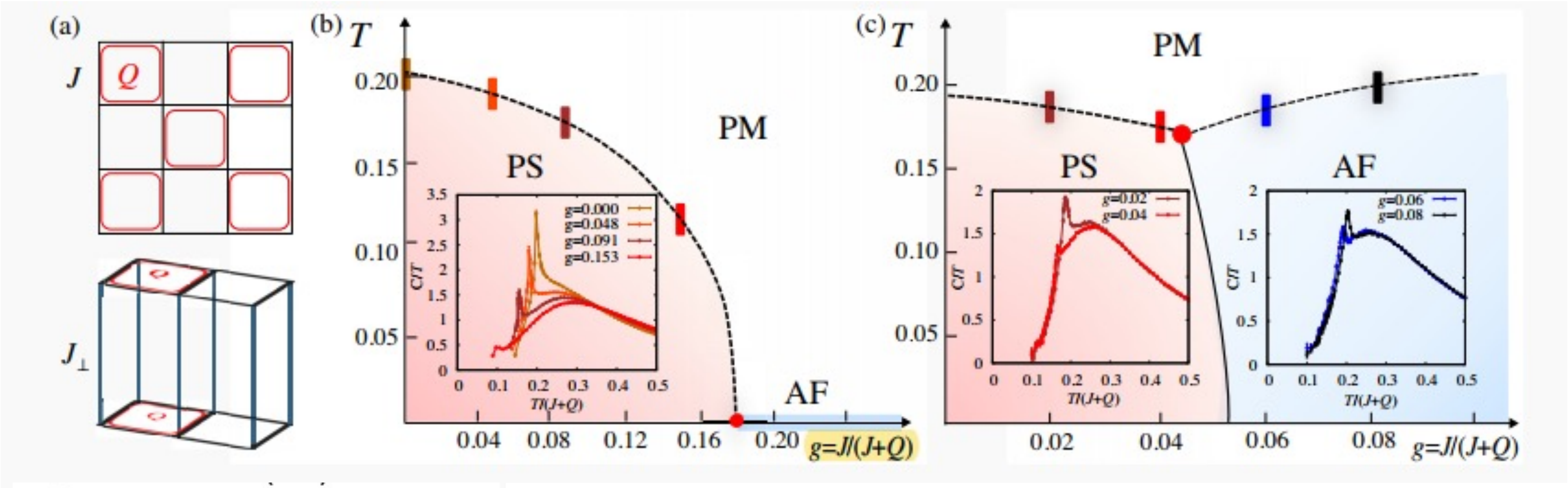
$$m_z = \frac{1}{L^2}\sum_{\mathbf{r}} \phi(\mathbf{r}) S^z(\mathbf{r}), \quad m_p = \frac{2}{L^2}\sum_{\mathbf{q}} \theta(\mathbf{q}) \Pi^z(\mathbf{q})$$

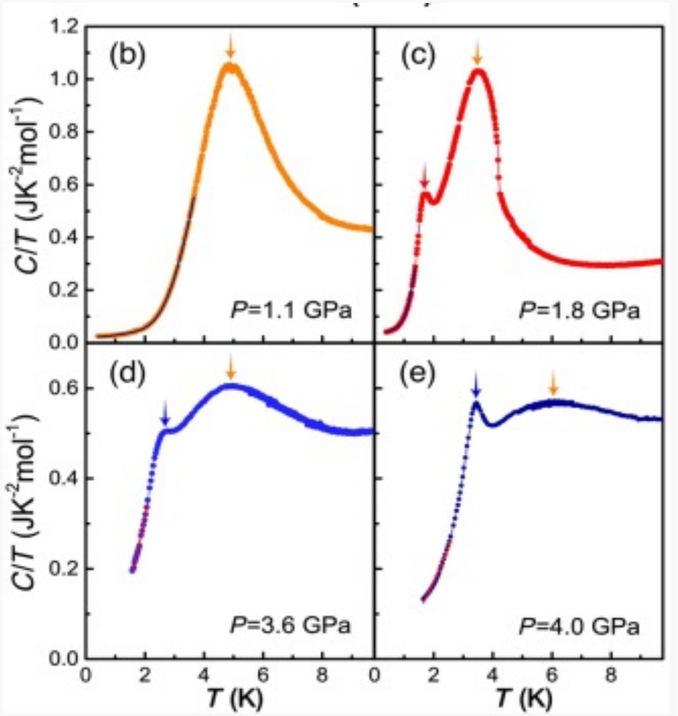
$$\Pi^z(\mathbf{q}) = S^z(\mathbf{q}) S^z(\mathbf{q}+\hat{x}) S^z(\mathbf{q}+\hat{y}) S^z(\mathbf{q}+\hat{x}+\hat{y})$$

$$U_p = \frac{3}{2}\left(1 - \frac{\langle m_p^4 \rangle}{3\langle m_p^2 \rangle^2}\right), \quad U_z = \frac{5}{2}\left(1 - \frac{\langle m_z^4 \rangle}{3\langle m_z^2 \rangle^2}\right)$$

Emergent O(4) symmetry

Bowen Zhao et al. Nat. Phys. 15 678 (2019)





J. Guo *et al.*, PRL 124 206602 (2020)

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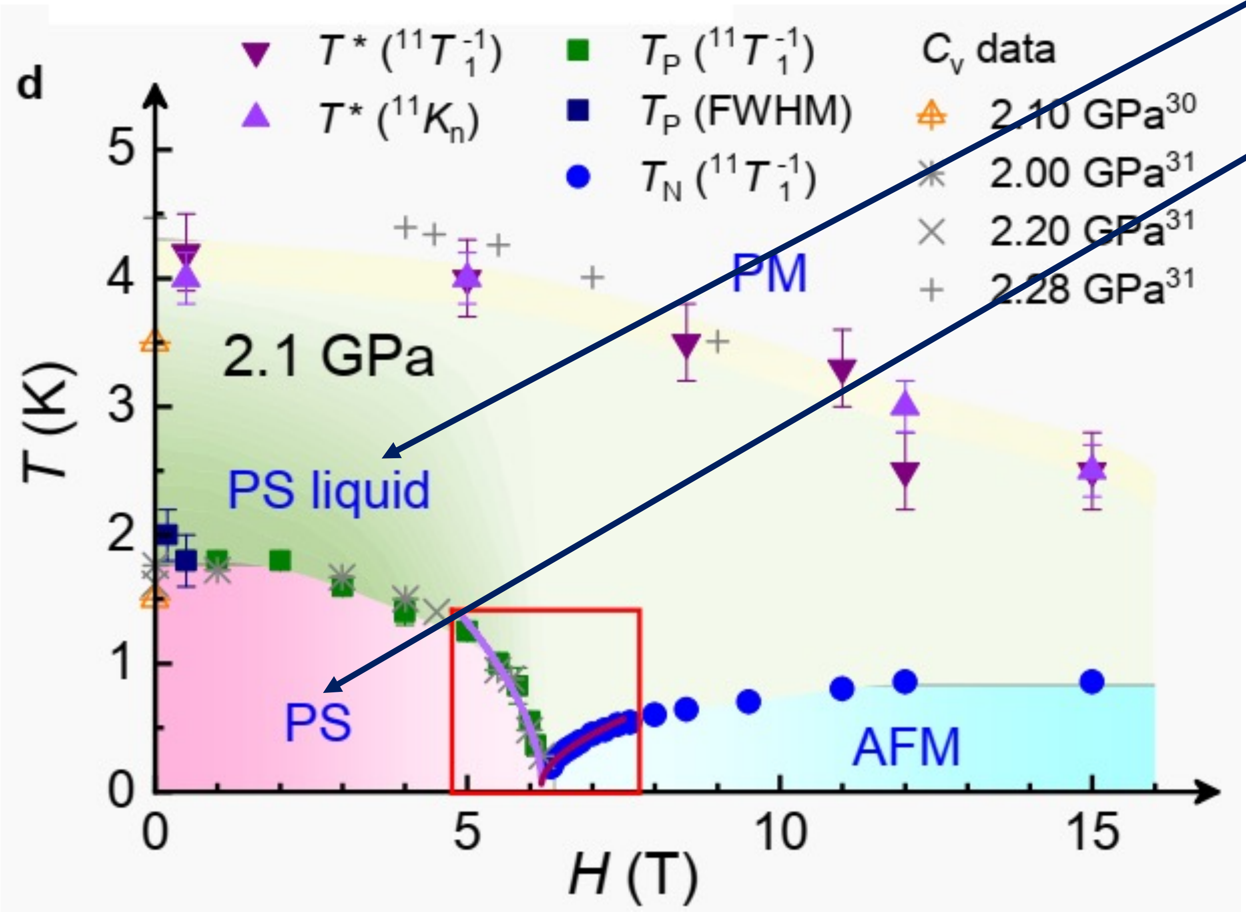
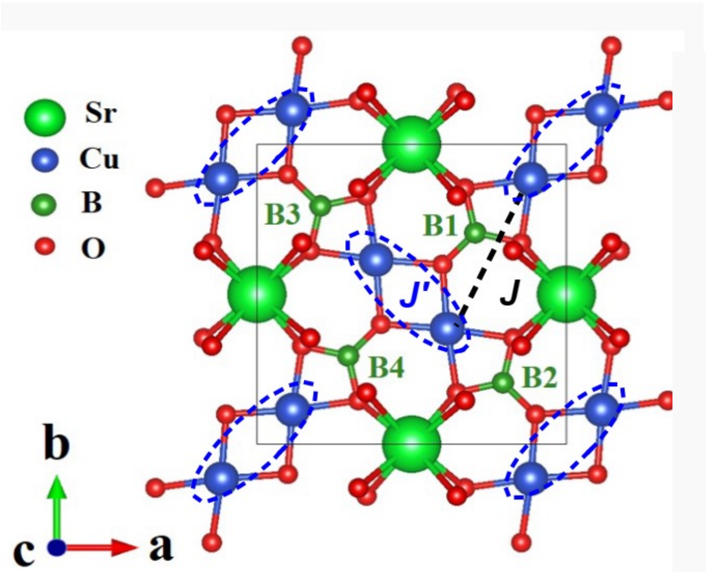
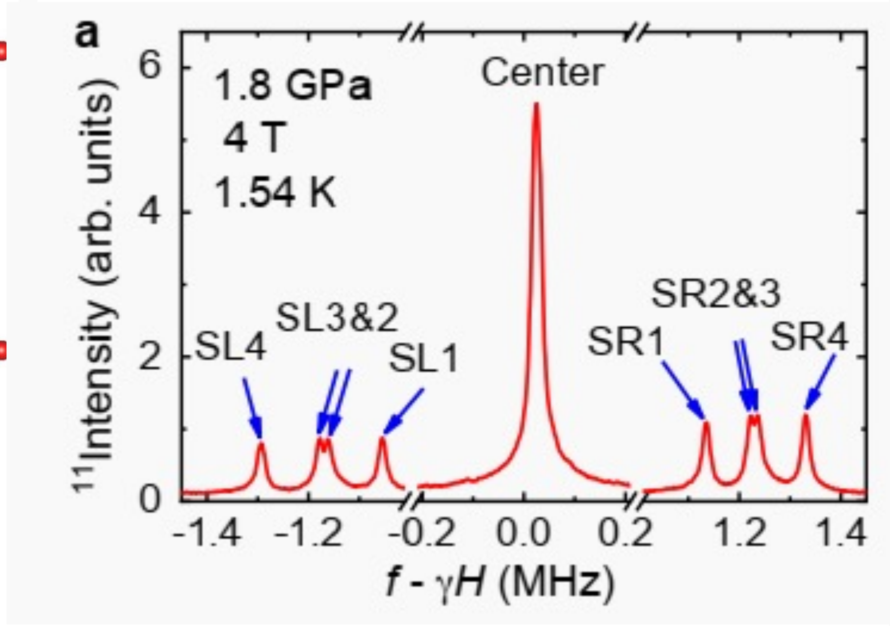
NMR results for $SrCu_2(BO_3)_2$

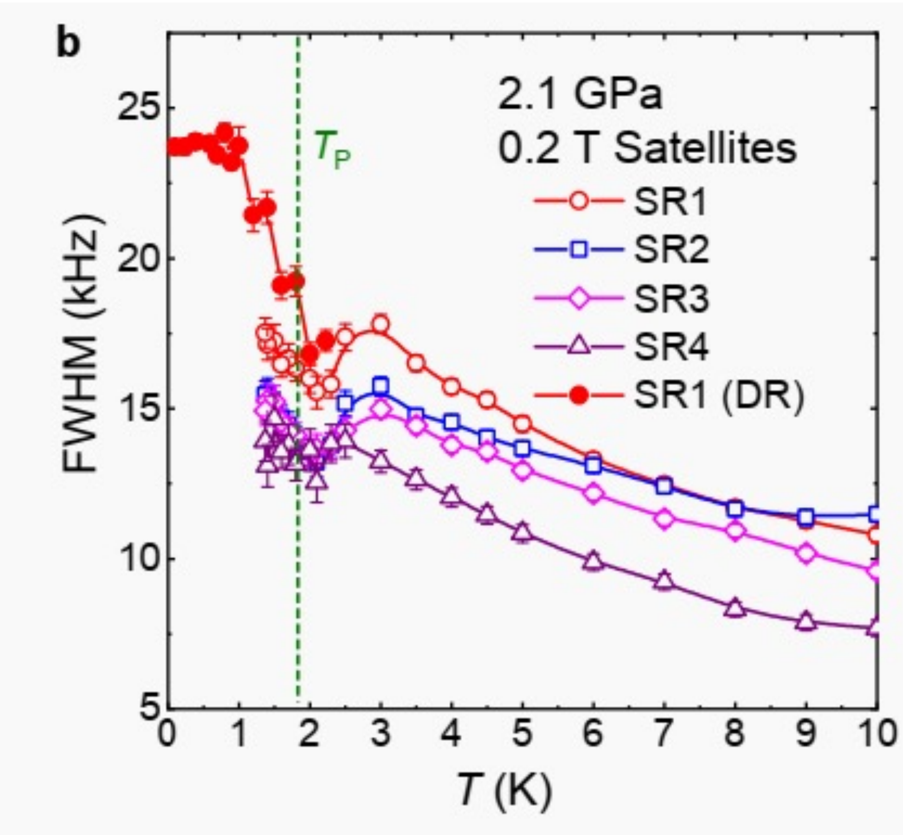
High-pressure $^{11}B\left(I = \frac{3}{2}\right)$ NMR



1. Plaquette liquid phase
2. Full-plaquette VBS phase
3. AFM ordered phase
4. A proximate DQCP between PVBS-AFM





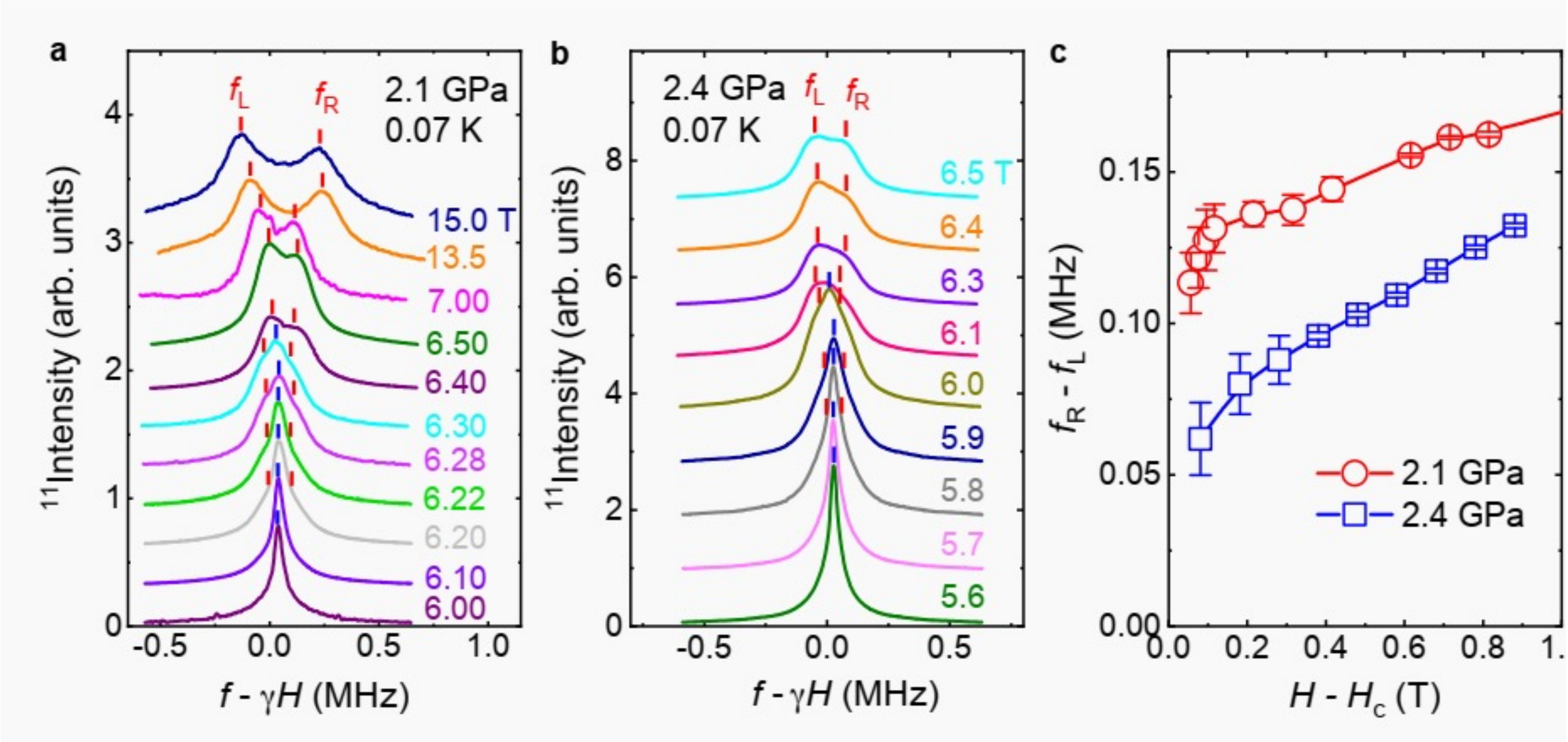


H is 8.6° away from c-axis
⇒ 4 inequivalent B sites

NMR spectrum

FWHM
(full-width at half maximum)
of satellites SR1 to SR4
T_P upturn tempearture

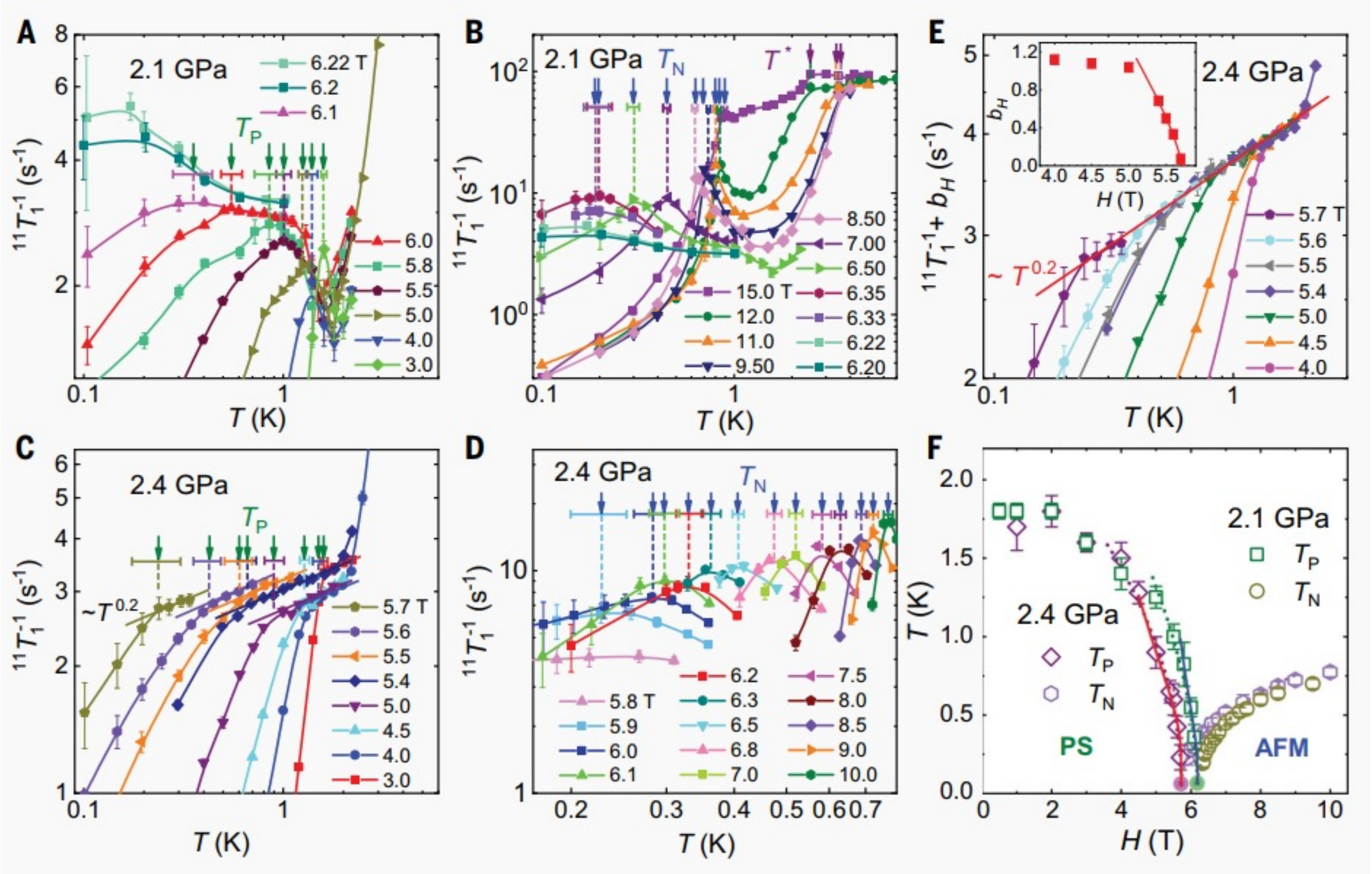
PS Liquid to PS

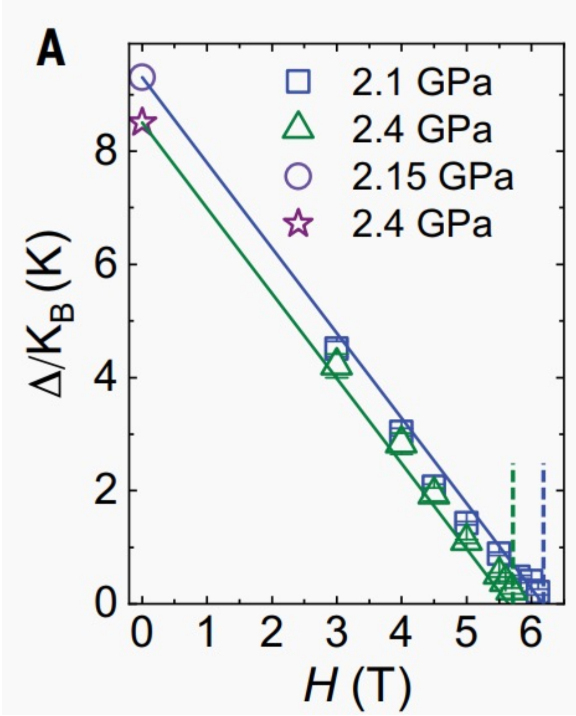


AFM transition: split of center line $f_R - f_L$ as order parameter

The much smaller first-order discontinuity of $f_R - f_L$ at the higher pressure indicates the approach toward a continuous QPT

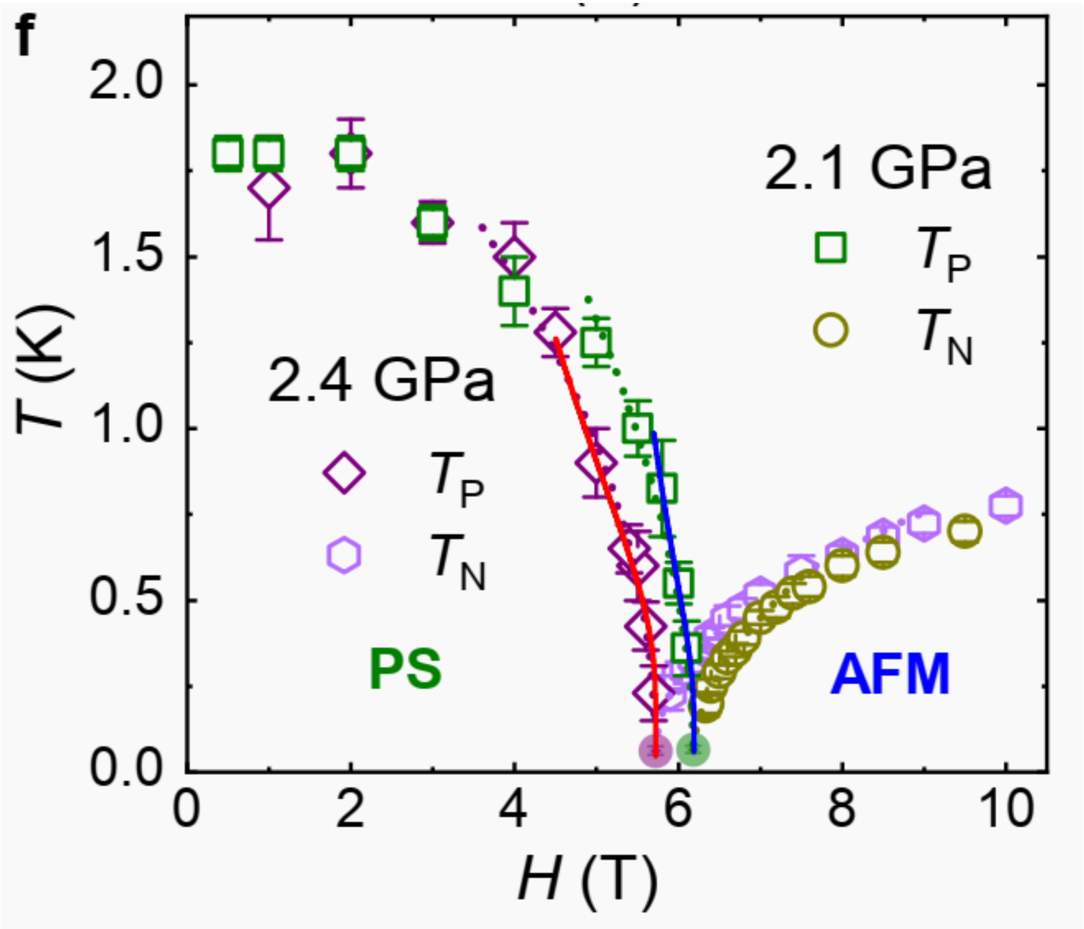
Spin-lattice relaxation rate $1/T_1$





expected from DQCP dualities

$$T_{P,N} = T_c + a_{P,N}|H - H_c|^{\phi}$$



2.1 GPa

$H_c = 6.184 \pm 0.05\ T$

$T_c = 0.066 \pm 0.010\ K$

$\phi = 0.569 \pm 0.025$

2.4 GPa

$H_c = 5.719 \pm 0.006\ T$

$T_c = 0.062 \pm 0.012\ K$

$\phi = 0.489 \pm 0.031$

$\phi = z\nu$

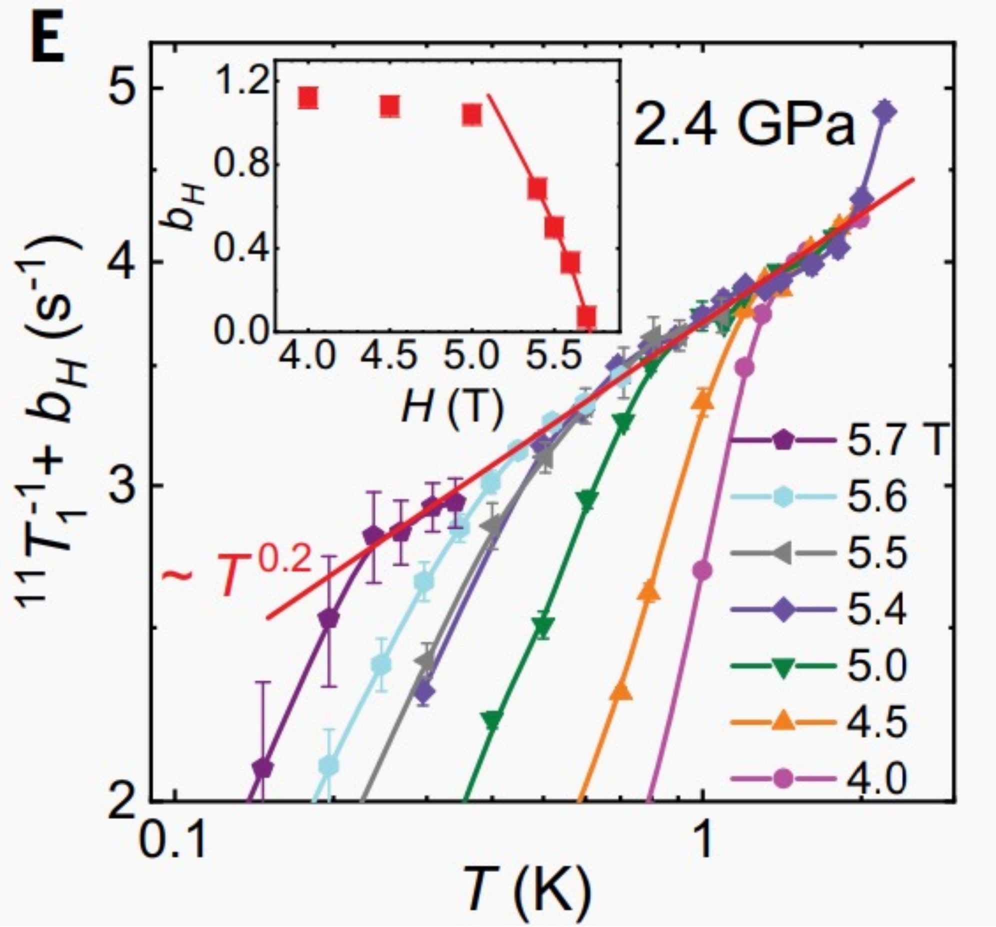
SO(5) $\phi = 0.46$ Nahum PRX 2015

Bowen PRL 2020

O(4) $\phi = 0.48$ Y. Qin PRX 2017

T_c small ⇒ a proximate DQCP

Duality ⇒ emergent symmetry to support DQCP



quantum-critical scaling of the spin-lattice relaxation at 2.4 GPa

$$\frac{1}{T_1} = aT^{\eta} + b_H$$

O(4) $\eta = 0.13(3)$ Y. Qin PRX (2017)

SO(5) $\eta = 0.26(3)$ A. W. Sandivk PRL (2007)

$\eta_{Neel} = 0.259(6), \eta_{VBS} = 0.25(3)$ Nahum PRX (2015)

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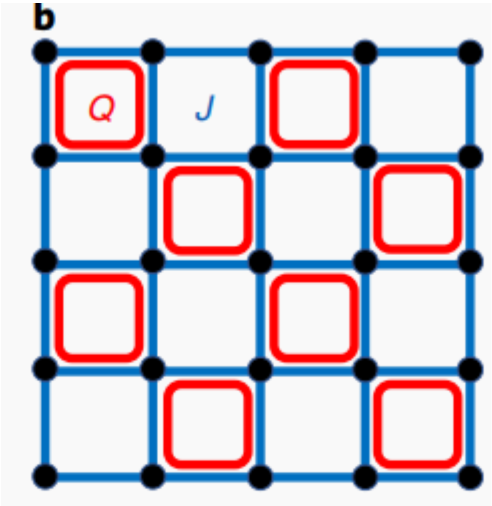
Numerical results for CBJQ model with magnetic field

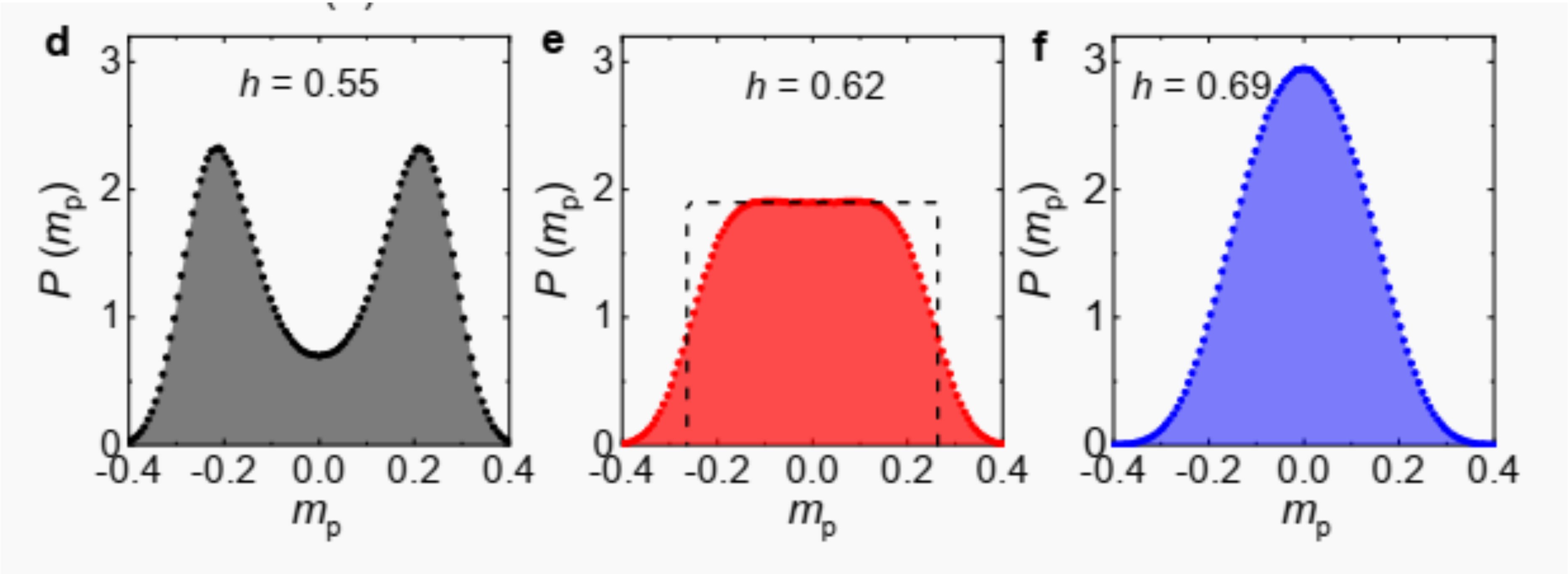


$$g_c = \frac{J}{Q} = 0.2175$$

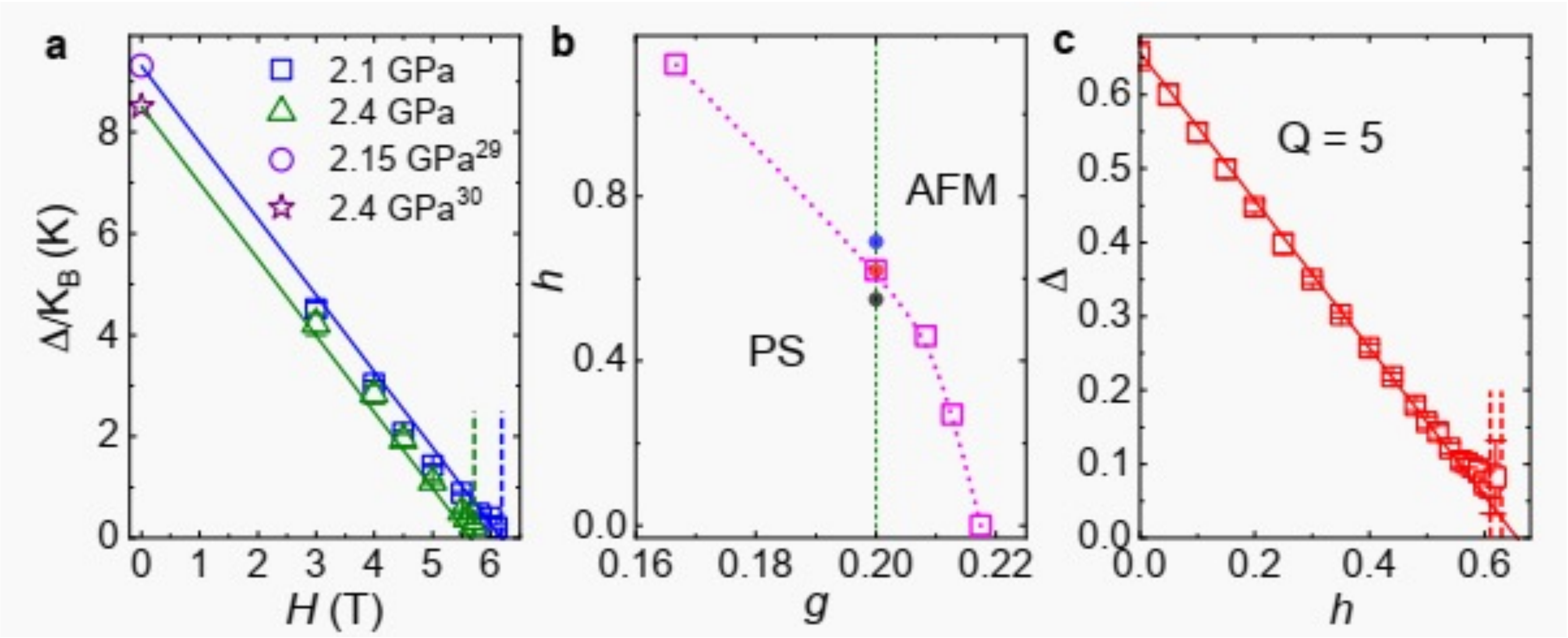
$$\mathcal{H} = -J\sum_{\langle ij\rangle} P_{ij} - Q\sum_{ijkl\in\Box_s}(P_{ij}P_{kl} + P_{ik}P_{jl}) - H\sum_i S_i^z$$

m_p order parameter of PS phase

Histogram of m_p at $g = \frac{J}{Q} = 0.2$ with $L = 32$



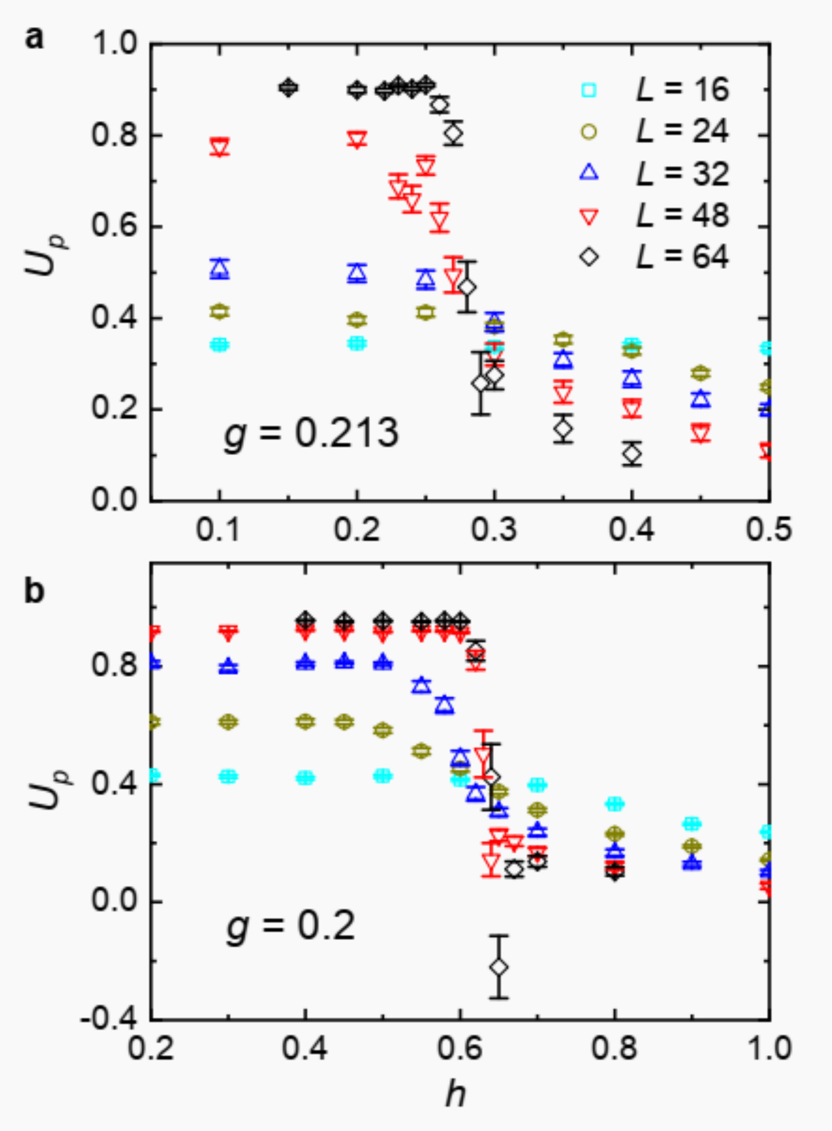
PS phase	At transition	AFM phase
Double peak	Uniform distribution	Single peak
Z_2 symmetry breaking	Emergent O(3) symmetry From U(1)*Z2	U(1) symmetry breaking



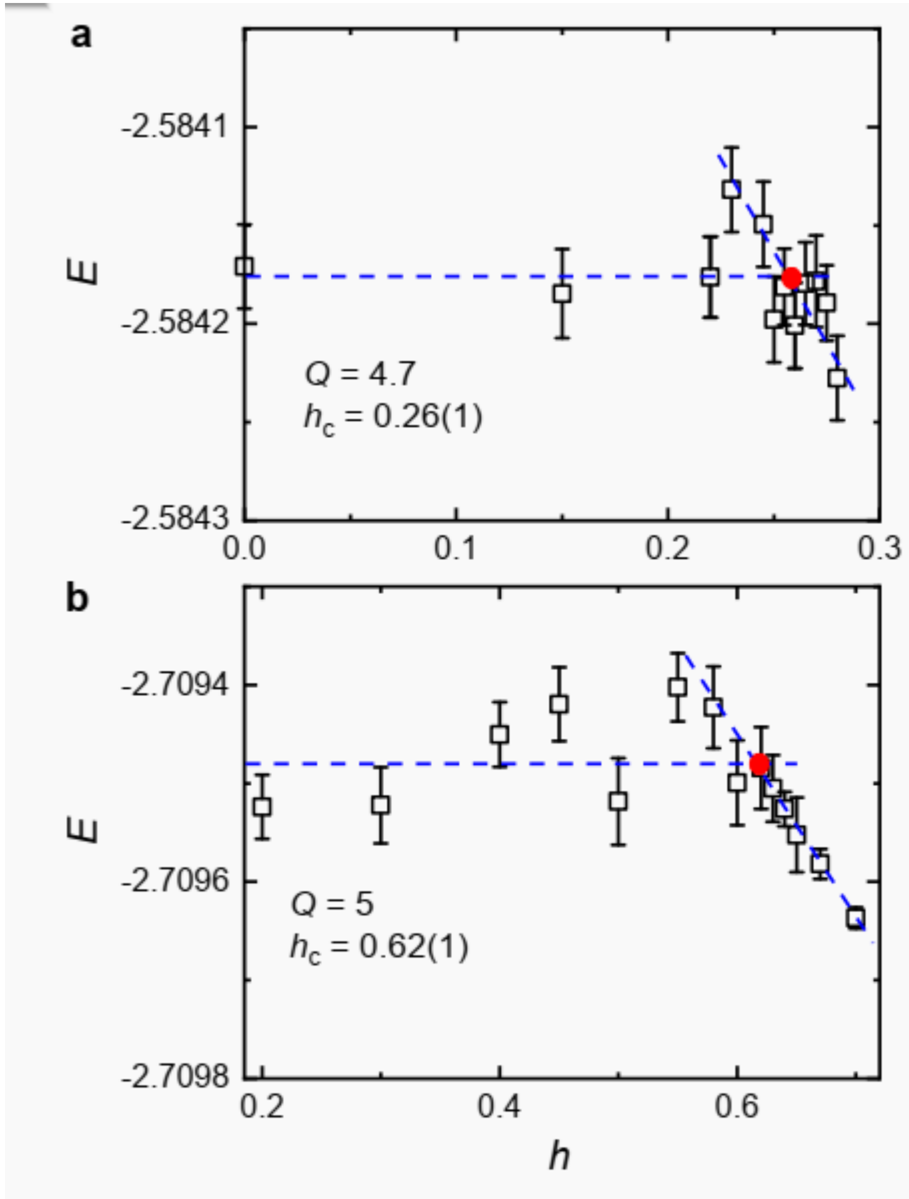
Experiment

CBJQ+h
phase diagram

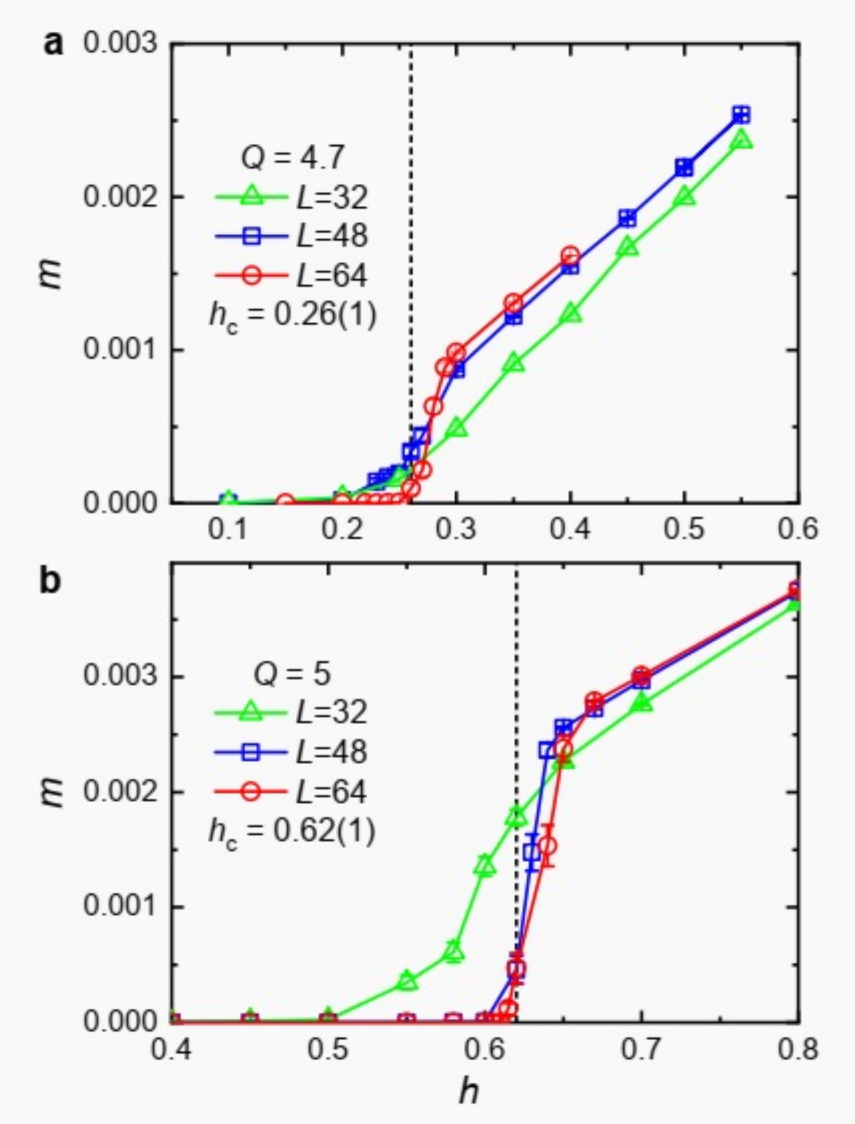
Simulation

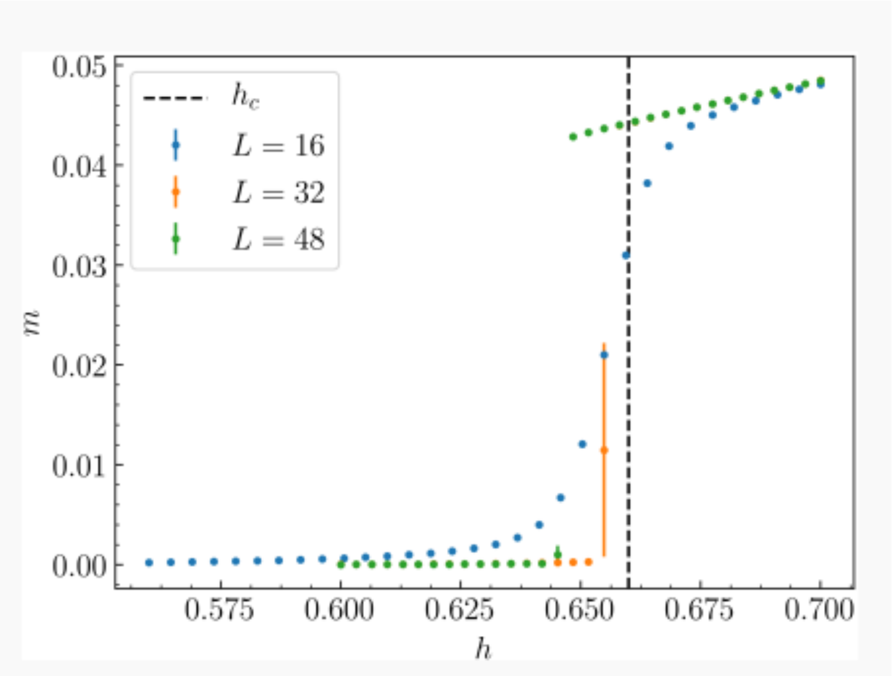


Close to g_c, first order transition become more and more weaker



$L = 48$





$$\mathcal{H} = -J\sum_{\langle ij\rangle}(P_{ij} - \lambda S_i^z S_j^z) - H\sum_i S_i^z,$$

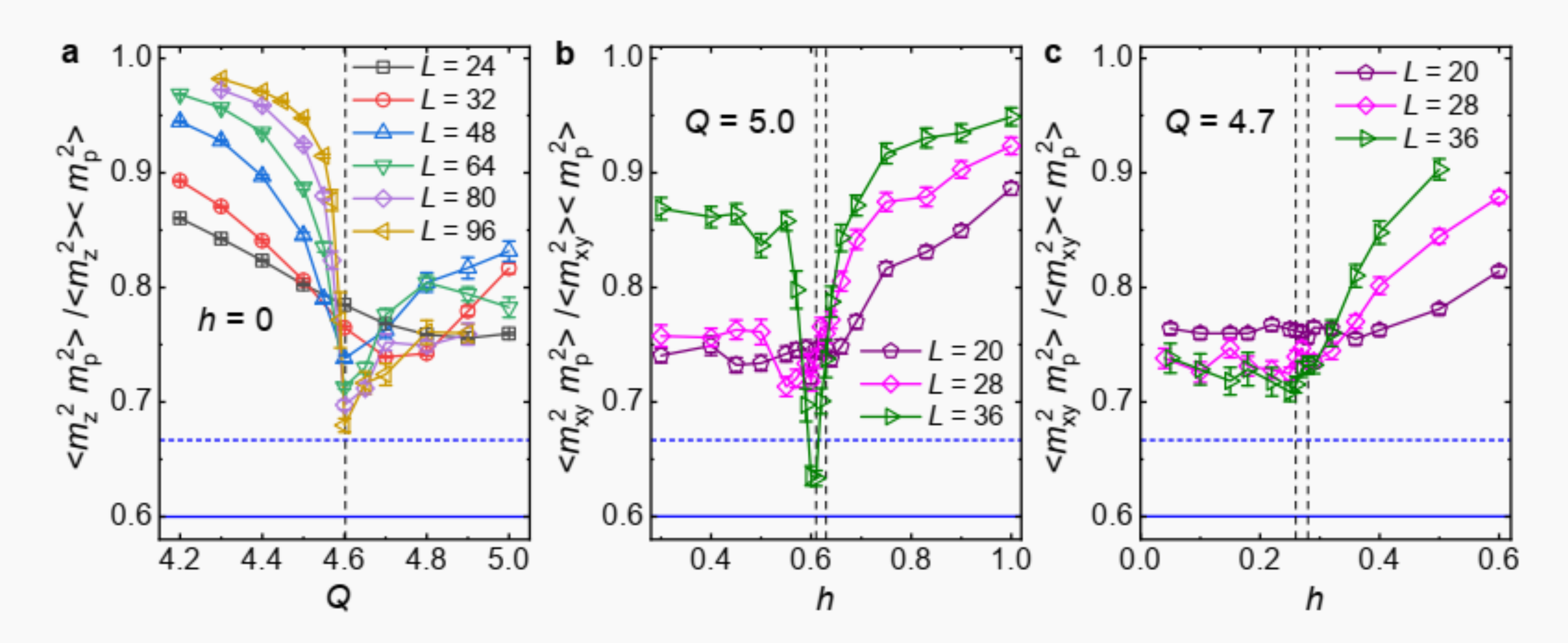
$\lambda = 0.1$

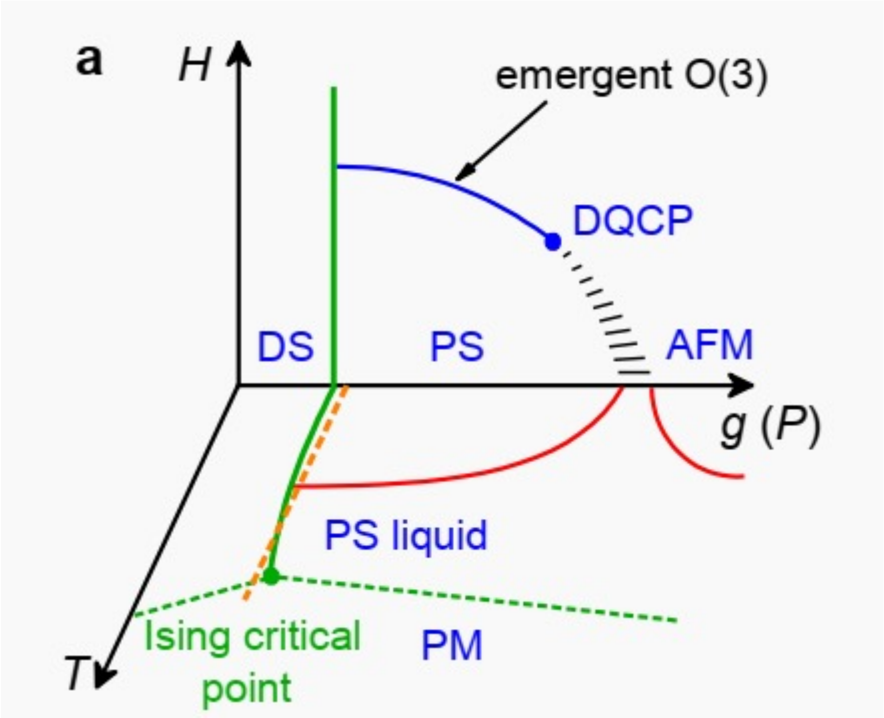
XXZ+h model m_z jumps 10^{-2}

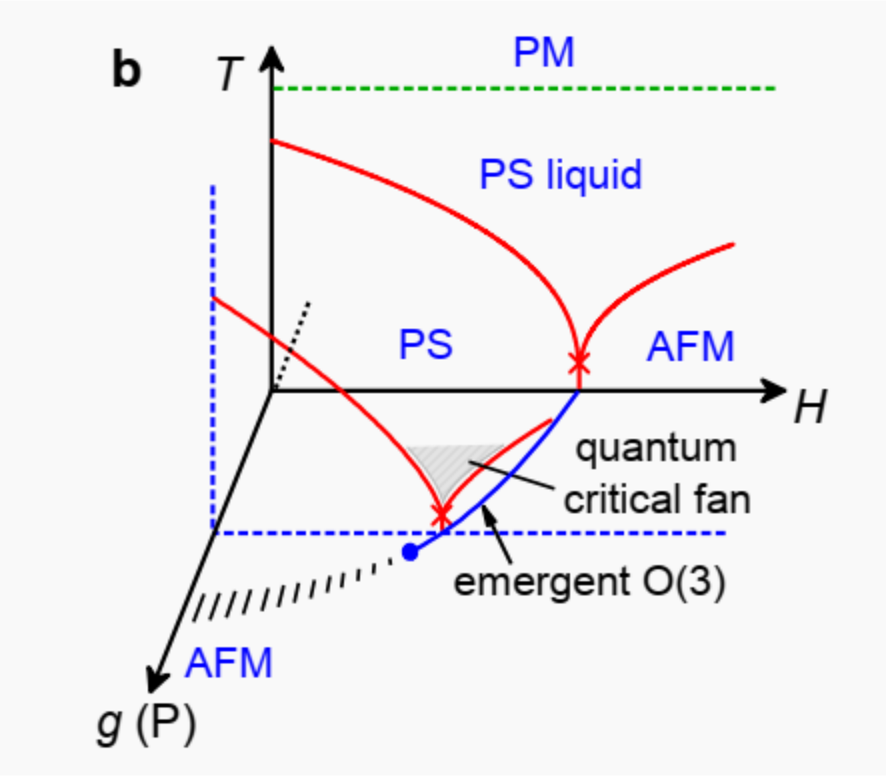
CBJQ+h m_z jumps 10^{-3}

cross-correlation ratio

$$m_{xy}^2 = \frac{1}{2}(m_x^2 + m_y^2) = \frac{1}{4N^2}\sum_{i,j}(-1)^{i-j}(S_i^+S_j^- + S_i^-S_j^+).$$







H=0 DQCP emergent O(4) symmetry

H≠0 DQCP emergent O(3) symmetry

Conclusion

NMR experiments find proximate DQCP in $SrCu_2(BO_3)_2$

first-order signatures weaken with increasing pressure

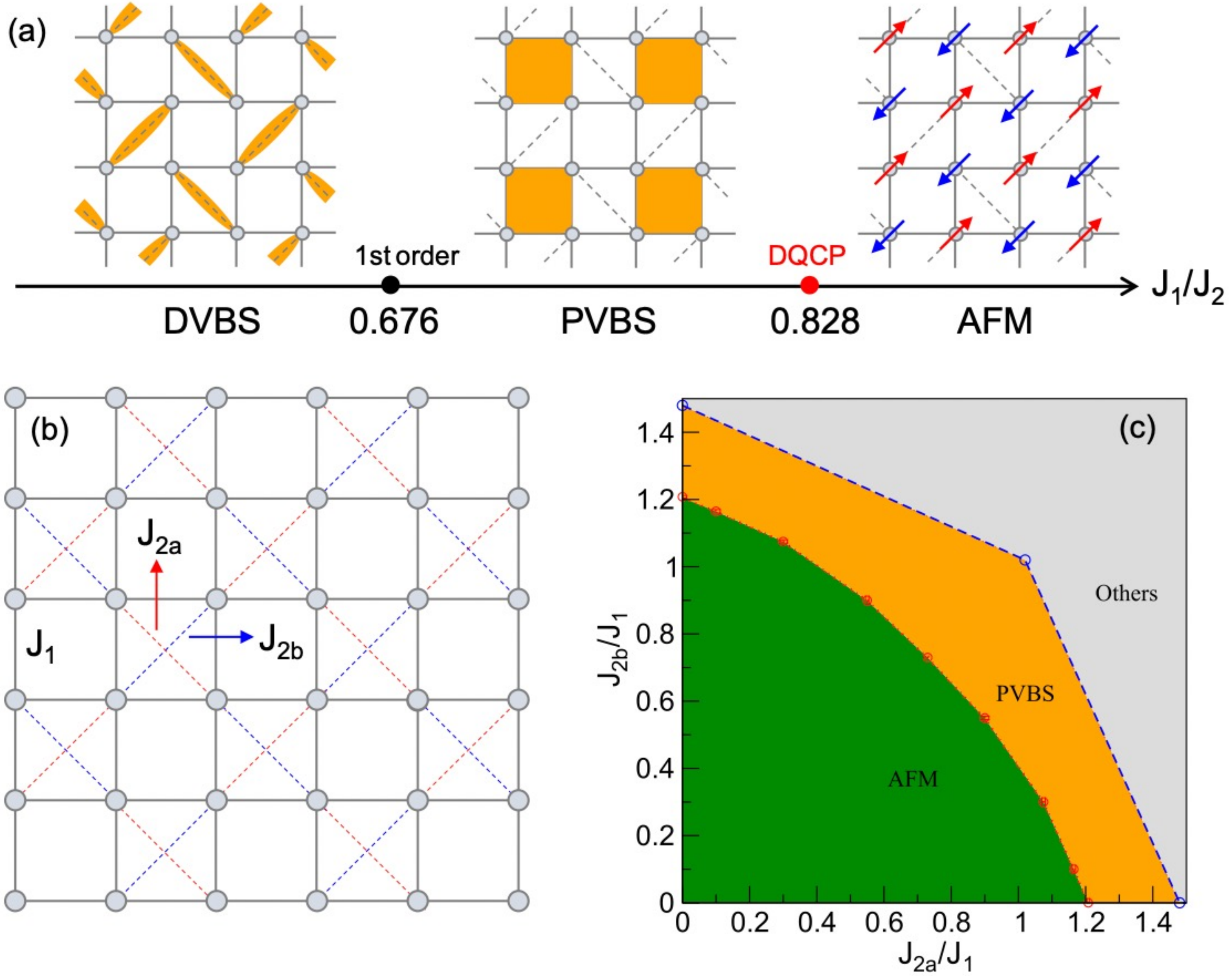
close to H_c, T_P *and* T_N are depressed to very low

energy gap and magnetization jump very small

quantum critical scaling at 2.4 GPa

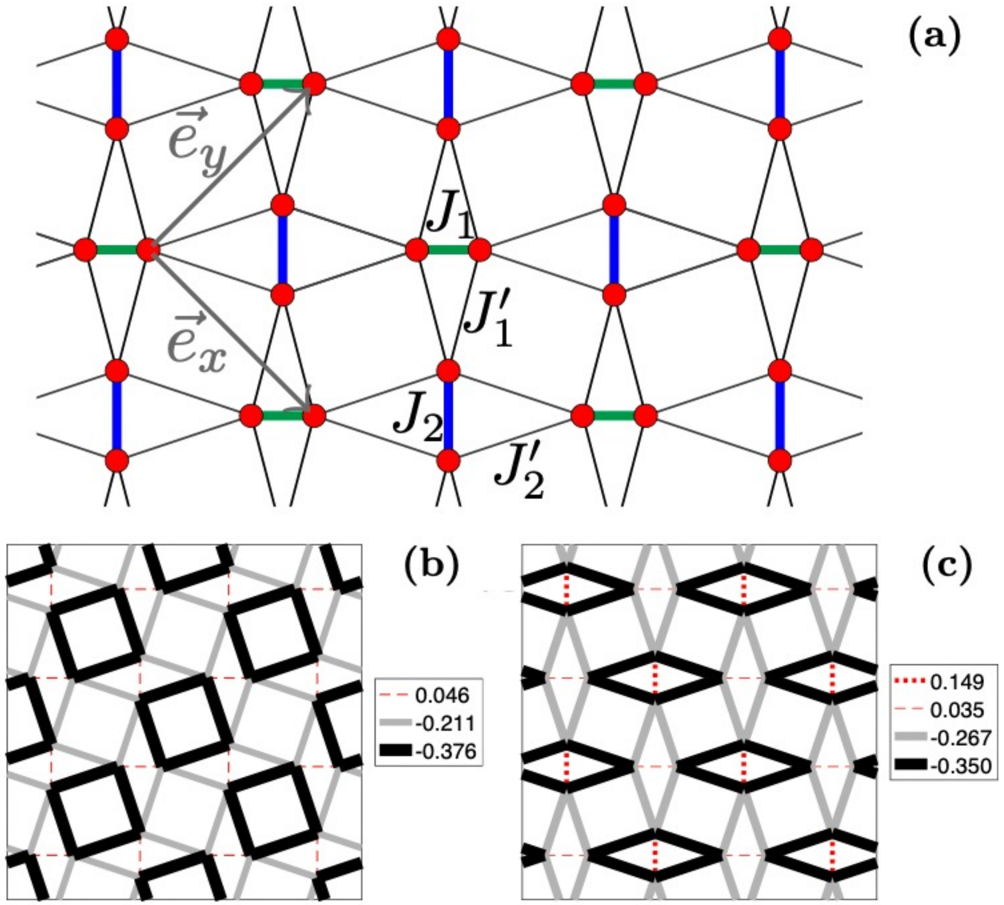
duality for the scaling of transition temperature

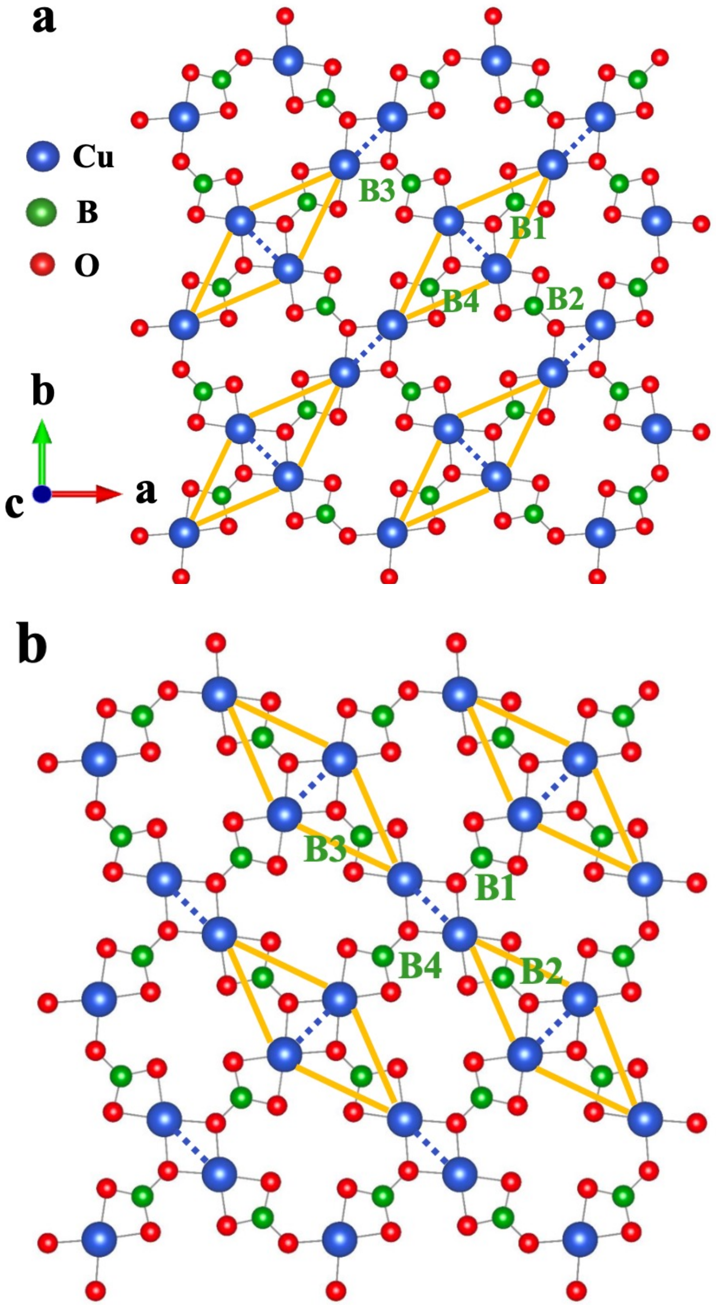
emergent O(3) symmetry



W. Liu, arXiv: 2309.10955(2023)

the finite projected entangled pair state (PEPS)





peak narrow

Line split broadening